MAA OMWATI INTERNATIONAL

EDUCATION CITY

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(HR.)

NOTES

BA 3RD SEM

Sub:- OPERATION RESEARCH

What is Operations Research ? -

1. OR is an art of winning the war without actually fighting it.

2. OR is the art of finding bad answers where worst exist. -T.L. Saaty (1958)

3. OR is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. -Morse and Kimbal (1946)

4. OR is a scientific method of providing executive with an analytical and objective basis for decisions.

5. OR is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of the operations with optimum solutions to the problem.
-Churchman, Acoff, Arnoff (1957)

Applications of OR

1. Finance, Budgeting and Investments :

(a) Claims and complaint procedures.
(b) Credit policies, credit risks and delinquent

account procedures.

(c) Cash-flow analysis, long range capital requirements, dividend policies, investment portfolios.

2. Purchasing, Procurement and Exploration:

(a) Rules for buying, supplies and ~~suitable~~ stable or varying prices.
(b) Determination of quantities and timing of purchases.
(c) Bidding policies:
(d) Strategies for exploration and exploitation of raw material sources.
(e) Replacement policies.

3. Production Management:

(i) Physical Distribution :- (a) location and size of warehouse, distribution centres and retail outlets.
(b) Distribution policy.

(ii) Facilities Planning :- (a) Numbers and location of factories, warehouses, hospitals etc.
(b) Loading and unloading facilities for railroads and trucks determining the transport schedule.

(iii) Manufacturing :- (a) Production scheduling and sequencing.
(b) Stabilization of production and employment training, layoffs and optimum product mix.

(iv) Maintainance and Project Scheduling :- (a) Maintenance policies and preventive maintenance.
(b) Maintenance crew size.
(c) Project scheduling and allocation of resources.

4. Marketing:

(a) Product selection, timing, competitive actions.
(b) Number of salesman, frequency of calling on accounts per cent of time spent on prospects.
(c) Advertising media with respect to cost and time.

5. Personnel Management:

(a) Selection of suitable personnel on minimum salary.
(b) Mixes of age and skills.
(c) Recruitment policies and assignment of jobs.

6. Research and Development :

(a) Determination of the areas of concentration of research and development.
(b) Project selection.
(c) Determination of time cost trade-off and control of development projects.
(d) Reliability and alternative design.

We can conclude that OR can be widely used in taking timely management decisions and also used as a corrective measure.

Main Characteristics of OR.

1. Inter-disciplinary team approach : In OR, the optimum solution is found by a team of scientists selected from various disciplines such as mathematics, statistics, economics, engineering, physics etc.

~~For~~ ~~e~~ An OR team required for a big organization may include a statistician, an economist, a mathematician, one or more engineers, a psychologist, and some supporting staff like computer programmers, etc.

2. Wholistic approach to the system : The most of the problems tackled by OR have the characteristic that OR tries to find the best (optimum) decisions relative to largest possible portion of the total organization. The nature of organization is essentially immaterial.

3. Imperfectness of solutions : By OR techniques, we cannot obtain perfect answers to our problems but, only the quality of the solution is improved from worse to bad answers.

4. Use of scientific research : OR uses techniques of scientific research to reach the optimum solution.

5. To optimize the total output : OR tries to optimize total return by maximizing the profit and minimizing the cost or loss.

MAIN PHASES of OR Study

Phase I : Formulating the Problem :- Before proceeding to find the solution of a problem, first of all one must be able to formulate the problem in the form of an appropriate model.

To do so, following information is required ;

(i) Who has to take the decision?
(ii) What are the objectives?

(iii) What are the ranges of controlled variables?
(iv) What are the uncontrolled variables that may affect the possible solutions?
(v) What are the restrictions or constraints on the variables?

Phase II: Constructing a mathematical model:
The second phase of the investigations is concerned with the reformation of the problem in an appropriate form which is convinient for analysis. The most suitable form for this purpose is to construct a mathematical model representing the system under study. It requires the identification of both static and dynamic structural elements.

A mathematical model should include the following three important basic factors:
(i) Decision variables and parameters.
(ii) Constraints or Restrictions
(iii) Objective function.

Phase III: Deriving the solutions from the model
This phase is devoted to the computation of those values of decision variables that maximize (or minimize) the objective function. Such solution is called an Optimal solution which is always in the best interest of the problem under consideration.

Phase IV : Testing the model and its solution (updating model)
After completing the model, it is once again tested as a whole for the errors if any. A model may be said to be valid if it can provide a reliable prediction of the system's performance.

Phase V: Controlling the solution
This phase establishes controls over the solution with any degree of satisfaction. The model requires immediate modification as soon as the controlled variables change significantly, otherwise the model goes out of control. As the conditions are constantly changing in the world, the model and the solution may not remain valid for a long time.

Phase VI: Implementing the solution
Finally, the test results of the model are implemented to work. This phase is primarily executed with the cooperation of Operations Research experts and those who are responsible for managing and operating the systems.

Scope of Operations Research.

1. In Agriculture: With the explosion of population and consequent shortage of food, every country is facing the problem of –
   (i) optimal allocation of land to various crops in accordance with the climatic conditions.
   (ii) optimum distribution of water from various resources like canal for irrigation purposes.

Thus there is a need of determining best policies under the prescribed restrictions. Hence a good amount of work can be done in this direction.

2. In Finance: In these modern times of economic crisis, it has become very necessary for every government to have a careful planning for the economic development of her country.
   OR-techniques can be fruitfully applied:
   (i) to maximize the per capita income with minimum resources.
   (ii) to find out the profit plan for the company.
   (iii) to determine the best replacement policies.

3. In Industry: OR is useful to the Industry ~~Distri~~ Director in deciding optimum allocation of various limited resources such as men, machines, materials, money, time etc to arrive at the optimum decision.

4. In Marketing: With the help of OR techniques a Marketing Administrator can decide –
   (i) where to distribute the products for sale so that the total cost of transportation etc is minimum.
   (ii) the minimum per unit sale price.
   (iii) the size of the stock to meet the future demand.
   (iv) how to select the best advertising media with respect to time, cost, etc.
   (v) how, when, and what to purchase at the minimum possible cost?

5. In Personnel Management: A personnel manager can use OR techniques:
   (i) to appoint the most suitable persons on minimum salary.
   (ii) to determine the best age of retirement for the employees.
   (iii) to find out the number of persons to be appointed on full time basis when the workload is seasonal (not continuous).

6. In Production Management: A production Manager can use OR techniques:
   (i) to find out the number and size of the items to be produced.
   (ii) In scheduling and sequencing the production run by proper allocation of machines.
   (iii) In calculating the optimum product mix.
   (iv) To select, locate and design the sites for the production plants.

7. In L.I.C. :- OR approach is also applicable to enable the L.I.C. offices to decide:
(i) what should be the premium rates for various modes of policies.
(ii) how best the profits could be distributed in the cases of with profit policies?

In Conclusion, we can say, wherever there is a problem, there is OR.

Linear Programming Problems [LPP]

The general LPP is for optimizing (maximizing / minimizing) a linear function of variables called the objective function subject to a set of linear equations and/or inequalities called the constraints or restrictions.

Eg: Max $z = 2x_1 + 3x_2$ } objective functions
s.t

$$\left.\begin{aligned} x_1 + x_2 &\leq 3 \\ 2x_1 + 5x_2 &\geq 7 \end{aligned}\right\} \text{set of constraints}$$

$x_1, x_2 \geq 0$ } non-negative restriction

Formulation of LPProblems

Ques1 A firm manufactures two-types of products A and B and sell them at a Profit of ₹2 on type A and ₹3 on type B. Each product is processed from two machines G & H. Type A requires 1 min of processing time on G and 2 min on H. Type B requires 1 min on G and 1 min on H. The machine G is available for not more than 6 hr 40 min, while machine H is available for 10 hr during any working day. Formulate the problem as a LPP.

Soln: Let x_1 be the no. of products of Type A & x_2 be the no. of products of Type B

(Total Profit) Max $z = 2x_1 + 3x_2$ } objective function

G = 400 min   H = 600 min

$$\text{s.t,}\quad \left.\begin{aligned} x_1 + x_2 &\leq 400 \\ 2x_1 + x_2 &\leq 600 \end{aligned}\right\} \text{set of constraints}$$

$x_1, x_2 \geq 0$ } non-ve restrictions.

Ques2 A company produces two types of Hats. Each hat of the first type requires twice as much labour time as the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second type to 150 and 250 hats. Assuming that the profits per hat are Rs. 8 for type A and Rs. 5 for type B, formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Solⁿ: Let x_1 be the hats of Type A.
& x_2 be the hats of Type B.

$$\text{Max } z = 8x_1 + 5x_2 \quad \} \text{ Objective function}$$

s.t

$$\left.\begin{aligned} 2x_1 + x_2 &\le 500 \\ x_1 &\le 150 \\ x_2 &\le 250 \end{aligned}\right\} \text{set of constraints}$$

$$x_1, x_2 \ge 0 \quad \} \text{ non -ve restrictions.}$$

Ques 3 The manufacturer of patent medicines is proposed to prepare a production plan for medicines A and B. There are sufficient ingredients available to make 20,000 bottles of medicines A and 40,000 bottles of medicine B, but there are only 45,000 bottles into which either of the medicines can be filled. Further it takes three hours to prepare enough material to fill 1000 bottles of medicine A and one hour to prepare enough material to fill 1000 bottles of medicine B, and there are 66 hours

available for their operation. The profit is Rs 8 per bottle for medicine A and Rs 7 per bottle for medicine B.

(i) Formulate this problem as an L.P.P.
(ii) How the manufacturer schedule his production in order to maximize profits.

Solⁿ: Let there be x_1 thousand bottles of medicine A and x_2 thousand bottles of medicine B

Profit on thousands bottles of x_1 is at the rate of Rs 8 and ~~on~~ thousand bottles of x_2 is at the rate of Rs 7.

$$\Rightarrow \cancel{\text{Max}}\ P = 8 \times 1000\, x_1 + 7 \times 1000\, x_2$$

$$\therefore \text{Max } P = 8000\, x_1 + 7000\, x_2 \quad \{\text{objective fxn}\}$$

Time required to fill x_1 thousand bottles of medicine A will be $3x_1$ and ~~x_2 thousand~~ medicine B will be x_2
~~But, total~~ $\Rightarrow 3x_1 + x_2$
Now, total time available for this operation is 66 hrs.

$$\text{s.t} \quad \left.\begin{aligned} 3x_1 + x_2 &\le 66 \\ x_1 + x_2 &\le 45 \\ x_1 &\le 20 \\ x_2 &\le 40 \end{aligned}\right\} \text{Set of constraints}$$

$$x_1, x_2 \ge 0 \quad \} \text{ non - negative constraint}$$

Ques 4. A toy company manufactures two types of doll, a basic version – doll A and a delux version – doll B. Each doll of type B takes twice as long to produce as one of type A and the company would have time to make a maximum of 2000 per day. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The delux version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs 3.00 and Rs 5.00 per doll, respectively on doll A and B, then how many of each doll should be produced per day in order to maximize the total profit. Formulate this problem.

Solⁿ: Let x_1 and x_2 be the number of dolls produced per day of type A and B, respectively.

Let the doll A require t hrs so that the doll B requires 2t hrs. So the total time to manufacture x_1 and x_2 dolls should not exceed 2,000t hrs.

$\therefore \; tx_1 + 2tx_2 \leq 2000t$

$\Rightarrow$ Max $P = 3x_1 + 5x_2$ (objective function)

s.t

$x_1 + 2x_2 \leq 2000$ (time constraint)

$x_1 + x_2 \leq 1500$ (Plastic constraint)

$x_2 \leq 600$ (dress constraint)

$x_1 \cancel{\geq}, x_2 \geq 0$ (non -ve constraint)

Ques 5. A firm manufactures 3 products A, B and C. The profits are Rs 3, Rs 2 and Rs 4 resply. The firm has 2 machines and below is the required processing time in minutes for each machine on each product:

		Product		
		A	B	C
Machine	G	4	3	5
	H	2	2	4

Machine G and H have 2,000 and 2,500 machine – minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but no more than 150 A's.

Setup an L.P. problem to maximize profit. Do not solve it.

Solⁿ: Let x_1, x_2, x_3 be the no. of products A, B, C resply

	$A(x_1)$	$B(x_2)$	$C(x_3)$	Machine minute
G	4	3	5	2000
H	2	2	4	2500
max. manufacture	100	200	50.	

Max $P = 3x_1 + 2x_2 + 4x_3$ [objective fxn]

s.t,

$4x_1 + 3x_2 + 5x_3 \leq 2000$

$2x_1 + 2x_2 + 4x_3 \leq 2500$

$100 \leq x_1 \leq 150$

$200 \cancel{\leq} x_2, \; 50 \leq x_3$

[set of constraints]

$x_1, x_2 \geq 0$ [non -ve constraints]

21/07/23

Graphical Method

It is used only for two variable LPP.

Step1 Consider each inequality constraint as an equation.

Step2 Plot each equation on the graph.

Step3 Shade the common region or feasible region. Every point on ~~this~~ line will satisfy the equation of the line. If the inequality constraint corresponding to that line is less than equal to '≤' then the region below the line and line in the first quadrant due to non-negativity of the variables is shaded.

The points lying in the common region will satisfy all the constraints simultaneously.

Step4 Find the value of z (objective function) at each corner point of the feasible region.

Step5 Find out the value of z max/min as per the problem.

Ques) Solve the LPP using the graphical method

$$\text{Max } z = 5x_1 + 3x_2$$
$$\text{s.t. } 3x_1 + 5x_2 \leq 15$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

Solⁿ: **Step1 :** Consider the inequality as eqⁿ

$3x_1 + 5x_2 = 15$ — ①

$5x_1 + 2x_2 = 10$ — ②

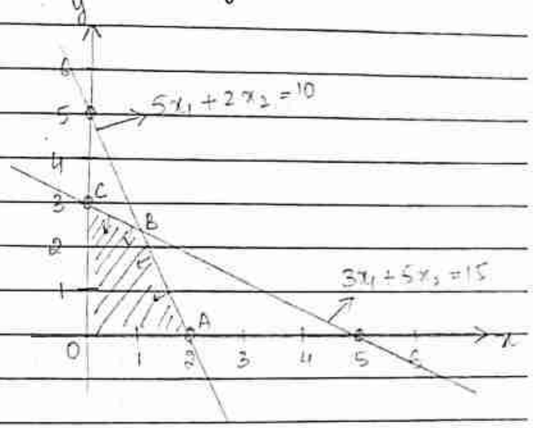
Step2: Plot the above eqⁿ on the graph

$3x_1 + 5x_2 = 15$

x_1	0	5	(0, 3)
x_2	3	0	(5, 0)

$5x_1 + 2x_2 = 10$

x_1	0	2	(0, 5)
x_2	5	0	(2, 0)



Step3 Shade the common region.
OABC is the feasible region.

Step4 Find the value of z at each point of the feasible region.

$\text{Max } z = 5x_1 + 3x_2$

$(0,0) \quad Z_O = 5(0) + 3(0) = 0$

$(2,0) \quad Z_A = 5(2) + 3(0) = 10$

(1.05, 2.36) $Z_B = 5(1.05) + 3(2.36)$
$= 5.25 + 7.08 = 12.33$.

(0, 3) $Z_C = 5(0) + 3(3) = 9$.

$5x_1 + 2x_2 = 10$
$x_2 = 5 - \frac{5}{2}x_1$

$3x_1 + 5x_2 = 15$
$3x_1 + 5\left(5 - \frac{5}{2}x_1\right)$
$3x_1 + 25 - \frac{25}{2}x_1 = 15$
$\frac{(6 - 25)}{2}x_1 = 15 - 25$

$\frac{19x_1}{2} = 10$
$\boxed{x_1 = \frac{20}{19} = 1.05}$
$\boxed{x_2 = 2.36}$

Step5: Find the max / min value of z.

Max. value of $z = 12.33$ at B.

Min value of $z = 0$ at 0.

$\therefore$ max. $z = \frac{235}{19}$, $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$.

Ques2 Solve the LPP using Graphical Method.

$$\text{Max } Z = 2x_1 + 3x_2$$

s.t.,
$x_1 + x_2 \geq 1$
$5x_1 - x_2 \geq 0$
$x_1 + x_2 \leq 6$
$x_1 - 5x_2 \leq 0$
$x_2 - x_1 \geq -1$
$x_2 \leq 3$
$x_1, x_2 \geq 0$

Solⁿ: **Step1:** Consider the inequality as eqⁿ.:

$x_1 + x_2 = 1$ — ①
$5x_1 - x_2 = 0$ — ②
$x_1 + x_2 = 6$ — ③
$x_1 - 5x_2 = 0$ — ④
$x_2 - x_1 = -1$ — ⑤
$x_2 = 3$. — ⑥

Step2 Plot the above equations on a graph.

$x_1 + x_2 = 1$

x_1	0	1	(0,1)
x_2	1	0	(1,0)

$5x_1 - x_2 = 0$

x_1	0	1	(0,0)
x_2	0	5	(1,5)

$x_1 + x_2 = 6$

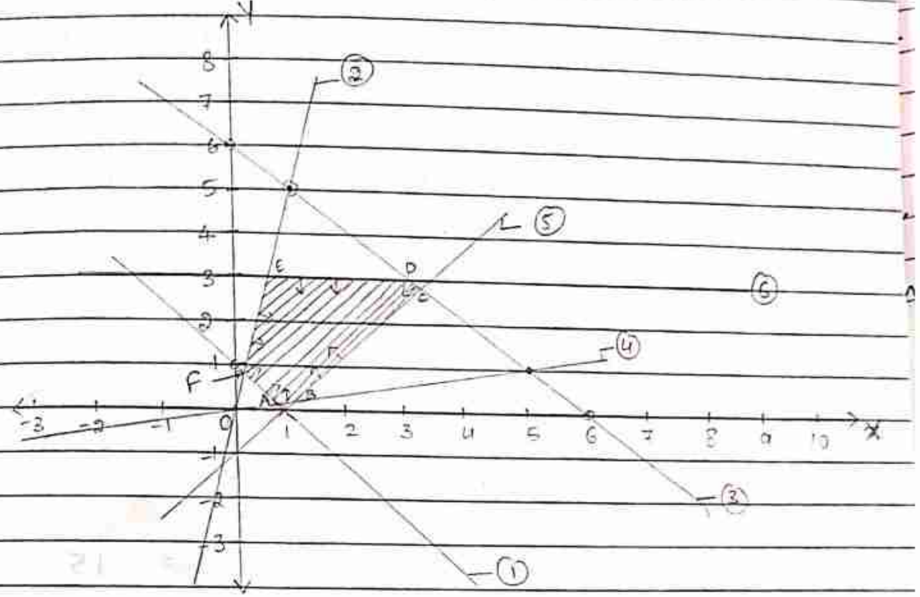
x_1	0	6	(0,6)
x_2	6	0	(6,0)

$x_1 - 5x_2 = 0$

x_1	0	5	(0,0)
x_2	0	1	(5,1)

$x_2 - x_1 = -1$

x_1	0	1	(0,-1)
x_2	-1	0	(1,0)



Step3. Shade the common region.
ABCDEF is the feasible region.

A → ① & ④ $x_1 + x_2 = 1$. $x_1 - 5x_2 = 0$ $A\left(\frac{5}{6}, \frac{1}{6}\right)$.

B → ④ & ⑤ $x_1 - 5x_2 = 0$ $x_1 - x_2 = 1$ $B\left(\frac{5}{4}, \frac{1}{4}\right)$

C → ③ & ⑤ $x_1+x_2=6$, $x_1-x_2=1$ $C\left(\frac{7}{2},\frac{5}{2}\right)$

D → ③ & ⑥ $x_1+x_2=6$, $x_2=3$ $D\left(3,3\right)$

E → ② & ⑥ $5x_1-x_2=0$, $x_2=3$ $E\left(\frac{3}{5},3\right)$

F → ① & ② $x_1+x_2=1$, $5x_1-x_2=0$ $F\left(\frac{1}{6},\frac{5}{6}\right)$

Step 4 Find the value of z at each point of the feasable region

$$\text{Max } z = 2x_1+3x_2$$

$\left(\frac{5}{6},\frac{1}{6}\right)$ $Z_A = 2\left(\frac{5}{6}\right)+3\left(\frac{1}{6}\right) = \frac{13}{6} = 2.16$

$\left(\frac{5}{4},\frac{1}{4}\right)$ $Z_B = 2\left(\frac{5}{4}\right)+3\left(\frac{1}{4}\right) = \frac{13}{4} = 3.25$

$\left(\frac{7}{2},\frac{5}{2}\right)$ $Z_C = 2\left(\frac{7}{2}\right)+3\left(\frac{5}{2}\right) = \frac{29}{2} = 14.5$

$(3,3)$ $Z_D = 2\left(3\right)+3\left(3\right) = 15 = 15$

$\left(\frac{3}{5},3\right)$ $Z_E = 2\left(\frac{3}{5}\right)+3(3) = \frac{51}{5} = 10.2$

$\left(\frac{1}{6},\frac{5}{6}\right)$ $Z_F = 2\left(\frac{1}{6}\right)+3\left(\frac{5}{6}\right) = \frac{17}{6} = 2.83$

Step 5 Find the max/min value of z

Max value of z = 15 at point D (3,3)

$x_1=3$
$x_2=3$

(minimization Problem)

Ques 3. Solve the LPP using Graphical Method.

$$\text{Min } z = 1.5x_1 + 2.5x_2$$

S.t $x_1 + 3x_2 \geq 3$
$x_1+x_2 \geq 2$
$x_1, x_2 \geq 0.$

Sol^n. **Step 1:** Consider the inequality of eq^n:

$x_1+3x_2=3$ —①
$x_1+x_2=2.$ —②

Step 2: Plot the above equations on a graph.

$x_1+3x_2=3$

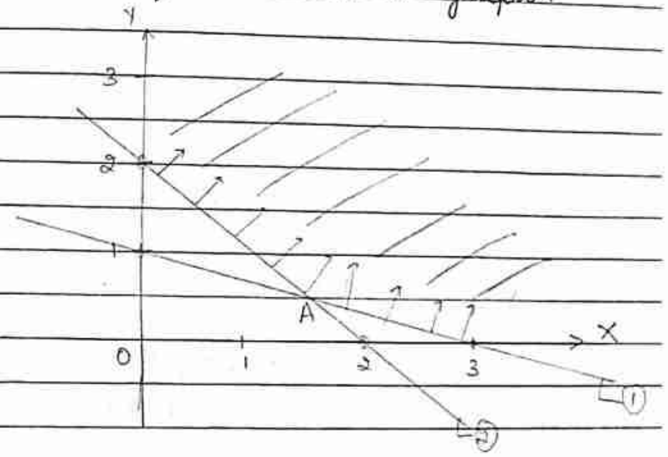
x_1	0	3
x_2	1	0

(0,1) (3,0)

$x_1+x_2=2$

x_1	0	2
x_2	2	0

(0,2) (2,0)



Step 3 Shade the common region.

A →
$$\begin{aligned} \cancel{x_1}+3x_2 &= 3 \\ \cancel{x_1}+x_2 &= 2 \\ - \quad - \quad & - \\ \hline 2x_2 &= 1 \end{aligned}$$

$\boxed{x_2=\frac{1}{2}}$

$x_1 = 2-\frac{1}{2} = \boxed{\frac{3}{2}}$

$A\left(\frac{3}{2},\frac{1}{2}\right)$

Step 4 Find the value of z at each point of feasible region.

$$Z_A = 1.5\left(\frac{3}{2}\right) + 2.5\left(\frac{1}{2}\right)$$

$$= \left(\frac{3}{2}\times\frac{3}{2}\right) + \left(\frac{5}{2}\times\frac{1}{2}\right) = \frac{9}{4} + \frac{5}{4} = \frac{14}{4} = 3.5$$

$$Z_A = 3.5$$

Step 5. The minimum value is attained at the point of intersection A

$$\min z = 3.5 \quad , \quad x_1 = \frac{3}{2} \quad , \quad x_2 = \frac{1}{2}.$$

(Problems having unbounded solution)

Ques 4 Solve the LPP using Graphical Method.

$$\text{Max } z = 3x_1 + 2x_2$$

S.t. $x_1 - x_2 \le 1$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0.$

ol^n: Consider the inequality of eq^n.

$x_1 - x_2 = 1$ — ①

$x_1 + x_2 = 3$ — ②.

$x_1 - x_2 = 1$

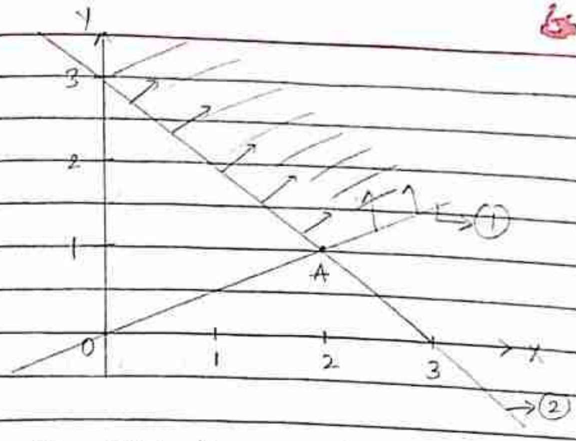
x_1	~~0~~1	2
x_2	0	1

(1,0) (2,1)

$x_1 + x_2 = 3$

x_1	0	3
x_2	3	0

(0,3) (3,0)



Here z can be made arbitrarily large and problem has no finite max. of z. Such problems are said to have unbounded solution

(Problems with inconsistent system of constraints)

Ques 5 Solve the LPP using Graphical Method.

$$\text{Max } z = 3x_1 - 2x_2.$$

S.t, $x_1 + x_2 \le 1$

$2x_1 + 2x_2 \ge 4$

$x_1, x_2 \ge 0.$

Sol^n: Consider the inequality of eq^n

$2x_1 + 2x_2 = 4$ — ②

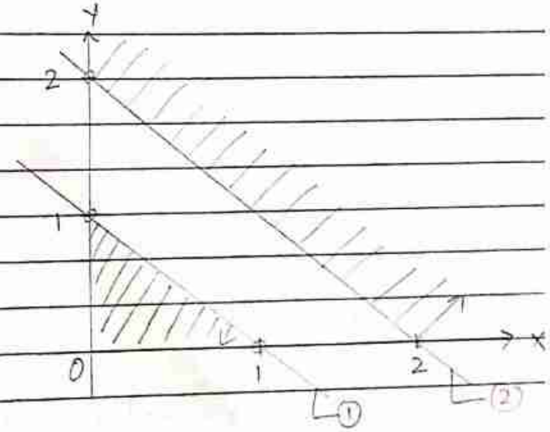
$x_1 + x_2 = 1$ — ①

$x_1 + x_2 = 1$

x_1	1	0
x_2	0	1

$2x_1 + 2x_2$

x_1	0	2
x_2	2	0



Here, there is no point (x_1, x_2) which satisfies both the constraints simultaneously. Hence, the problem has no solution because the constraints are inconsistent.

(Problems in which constraints are equations rather than inequalities.)

Ques6 Solve the LPP using Graphical Method.

$$\text{Max } z = 5x_1 + 3x_2$$

$$\text{s.t.,} \quad 3x_1 + 5x_2 = 15 \quad \text{—— ①}$$

$$5x_1 + 2x_2 = 10 \quad \text{—— ②}$$

$$x_1, x_2 \geq 0.$$

Sol^n:

$3x_1 + 5x_2 = 15$

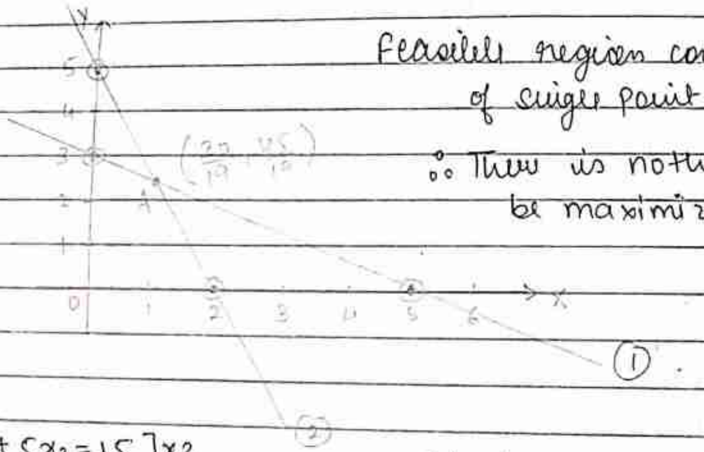
x_1	0	5
x_2	3	0

(0,3), (5,0)

$5x_1 + 2x_2 = 10$

x_1	0	2
x_2	5	0

(0,5), (2,0)



Feasible region consists of single point A.

∴ There is nothing to be maximized.

$$3x_1 + 5x_2 = 15 \,]\times 2$$
$$5x_1 + 2x_2 = 10 \,]\times 5$$

$$\Rightarrow 6x_1 + \cancel{10x_2} = 30$$
$$25x_1 + \cancel{10x_2} = 50$$
$$19x_1 = 20 \Rightarrow \boxed{x_1 = \frac{20}{19}}$$

$$3\left(\frac{20}{19}\right) - 15 = -5x_2$$

$$5x_2 = 15 - \frac{60}{19}$$

$$x_2 = 3 - \frac{12}{19}$$

$$\boxed{x_2 = \frac{45}{19}}$$

Iso-Profit or Iso-Cost Method

In this method, all the previous methods steps are used, then plot the line z=0 & the draw lines parallel to z=0.

Ques Find the solution of LPP:

$$\text{Max } z = 45x_1 + 80x_2$$

$$\text{s.t.} \quad 5x_1 + 20x_2 \leq 400$$

$$10x_1 + 15x_2 \leq 450$$

$$x_1, x_2 \geq 0.$$

Sol^n: Consider the inequality of eq^n

$$5x_1 + 20x_2 = 400 \quad \text{—— ①}$$

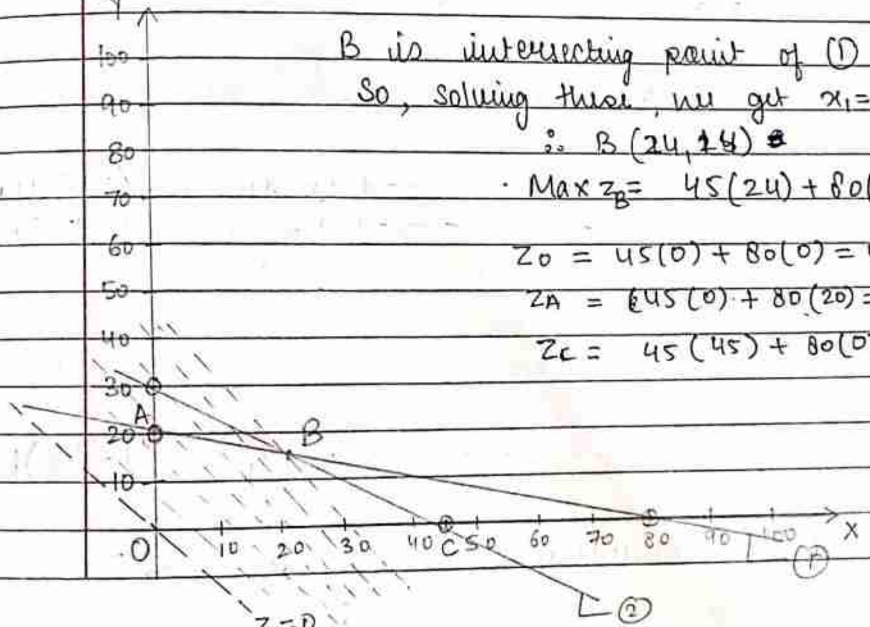
$$10x_1 + 15x_2 = 450 \quad \text{—— ②}$$

$5x_1 + 20x_2 = 400$

x_1	0	80
x_2	20	0

$10x_1 + 15x_2 = 450$

x_1	0	45
x_2	30	0



B is intersecting point of ① & ②.

So, solving these, we get $x_1 = 24$, $x_2 = 14$

∴ B(24, 14)

$$\text{Max } z_B = 45(24) + 80(14) = 2200$$

$$z_O = 45(0) + 80(0) = 0$$

$$z_A = 45(0) + 80(20) = 1600$$

$$z_C = 45(45) + 80(0) = 2025$$

Here, there is no point (x_1, x_2) which satisfies both the constraints simultaneously. Hence the problem has no solution because the constraints are inconsistent.

After plotting ||el lines to $z=0$, B is the last point which lies on the ||el lines
∴ B is the max. point.

Similarly, if we have to get min. point, then its the first point which lies on the ||el line drawn to $z=0$

General Linear Programming Problem

The general LPP having 'n' decision variables $x_1, x_2, ----, x_n$ to maximize / minimize the objective function

$$z = c_1x_1 + c_2x_2 + c_3x_3 + -----+ c_nx_n.$$

Subject to m-constraints :

$$a_{11}x_1 + a_{12}x_2 + -------+ a_{1n}x_n \quad (\leq / = / \geq) b_1$$
$$a_{21}x_1 + a_{22}x_2 + -------+ a_{2n}x_n \quad (\leq / = / \geq) b_2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + -------+ a_{mn}x_n \quad (\leq / = / \geq) b_m$$

& non-ve restrictions, $x_1, x_2, ----, x_n \geq 0$

Slack Variables : If a constraints has $\leq$ sign, then in order to make it an equality, we have to add something positive to the left hand side.

The non-negative variable which is added to the left hand side of the constraint to convert it into equation is called the slack variable.

Eg: Consider the constraints :

$$x_1 + x_2 \leq 2$$
$$2x_1 + 4x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

We add the slack variables $x_3 \geq 0$, $x_4 \geq 0$ to the left hand sides of the above equations respectively to obtain,

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 4x_2 + x_4 = 5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Surplus Variables : If a constraint has $\geq$ sign, then in order to make it an ~~in~~equality, we have to subtract something non-negative from its LHS.

Thus the positive variable which is subtracted from the LHS of the constraints to convert it into equation is called the surplus variable.

Eg: Consider the constraints :

$$x_1 + x_2 \geq 2$$
$$2x_1 + 4x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

we subtract the surplus variables $x_3 \geq 0, x_4 \geq 0$ from the left hand side of above inequalities, to obtain

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 4x_2 - x_4 = 5$$
$$x_1, x_2, x_3, x_4 \geq 0 .$$

The canonical form : The general formulation of LPP can always be expressed in the form :

$$\text{Max } z = c_1 x_1 + c_2 x_2 + ---- + c_n x_n$$

s.t.c

$$a_{i1} x_1 + a_{i2} x_2 + --- + a_{in} x_n \leq b_i \;;\; i = 1, 2, -m$$
$$\& \quad x_1, x_2, ---, x_n \geq 0 .$$

by making use of some elementary transformations. This form of LPP is called the canonical form of LPP.

The Standard Form :

tep1: All the constraints should be converted into equations except for the non-negativity restrictions which remains as inequalities (≥ 0).

2: The RHS element of each constraint should

Eg: $3x_1 - 4x_2 \geq -4 \Rightarrow 3x_1 - 4x_2 - x_3 = -4 \Rightarrow -3x_1 + 4x_2 + x_3 = 4$

be made non-negative (if not).

Step3: All variables must have non-negative values.

Step4: The objective function should be of maximizat form.

$$\text{Min } f(x) = -\text{Max}[-f(x)].$$
$$\text{Min } z = c_1 x_1 + c_2 x_2 + ---- + c_n x_n$$
$$\text{Max } z' = -c_1 x_1 - c_2 x_2 ------ c_n x_n$$
$$\text{with } z = -z'$$

Some Important Definitions

1. Solution to LPP : Any set $X = \{x_1, x_2, ---, x_{n+m}\}$ of variables is called a solution to LPP, if it satisfies the set of constraints only.

2. Feasible Solution (FS) : Any set $X = \{x_1, x_2, ---, x_n$ of variables is called a feasible solution (or programme) of L.P. problem, if it satisfies the set of constraints and non-negativity restrictions.

3. Basic Solution (BS) : A basic solution to the set of constraints is a solution obtained by setting any n variables (among $m+n$ variables) equal to zero and solving for remaining m variables provided the determinant of the coefficient of these m variables is non-zero. Such m variables are called basic variables

and remaining n zero-valued variables are called non-basic variables.

The no. of basic solutions thus obtained will be at the most

$$^{m+n}C_m = \frac{(m+n)!}{n!\,m!}$$

which is the no. of combinations of n+m things taken m at a time.

4. **Basic Feasible Solution (BFS) :** A basic feasible solution is a basic solution which also satisfies the non-negativity restrictions.

Basic feasible solutions are of two types in nature :

(i) Non-degenerate BFS :- A non-degenerate basic feasible solution is the basic feasible solution which has exactly m positive x_i $(i=1,2,---m)$.

OR

All m basic variables are positive and the remaining n variables will be zero.

(ii) Degenerate BFS :- A basic feasible solution is called degenerate, if one or more basic variables are zero valued.

5. **Optimum Basic Feasible Solution :** A BFS solution is said to be optimum, if it also optimizes (maximizes or minimizes) the objective function.

6. **Unbounded Solution :** If the value of the objective function z can be increased or decreased indefinitely, such solutions are called unbounded solutions.

Ques Convert the given LPP into standard form

$$\text{Min } z = 2x_1 - 3x_2$$

$$\text{s.t} \quad 2x_1 + x_2 \geq -4$$

$$x_1 + x_2 \leq 7$$

$$x_1, x_2 \geq 0$$

Sol^n: To convert the given LPP into standard form.

Use the slack variables

$$\text{Max } z' = -2x_1 + 3x_2 \qquad \text{where } z' = -z$$

$$\text{s.t} \quad -2x_1 - x_2 \leq 4$$

$$x_1 + x_2 \leq 7$$

$$-2x_1 - x_2 + s_1 = 4$$

$$x_1 + x_2 + s_2 = 7$$

$$x_1, x_2, s_1, s_2 \geq 0.$$

4/Aug/23

SIMPLEX METHOD

Simplex method examines corner points of the feasible region, using matrix row operations, until an optimal solution is found.

Slack variable represents an unused quantity of resource; it is added to less-than or equal-to type constraints in order to get an equality constraints.

Surplus variable represents the amount of resource usage above the minimum required and is added to greater-than or equal-to constraints in order to get equality constraint.

Basic feasible solution: A basic solution to the system of simultaneous equations is called basic feasible solution if $x_B \geq 0$.

C_B : coefficient of basic variable in objective function.

C_j : coefficient of variables in objective function or decision variable.

Degeneracy arises when there is a tie in the minimum ratio value that determines the variable to enter into the next sol^n.

Basis is the set of variables present in the solution, have positive non-zero value; these variables are also called basic variables.

Non-basic variables are those which are not in the 'basis' and have zero-value.

Infeasibility is the solution in which no solution satisfies all constraints of an LP problem.

$\Delta j = Cj - Zj$ row values represent net profit or loss resulting from introducing one unit of any variable into the 'basis' or solution mix.

Key column is in which variable has highest the value corresponding Δj.
Key row is in which minimum ratio has min value.
Key element is the point where key row and key column intersect.

Ques1 Use simplex method to solve the following LP problem.

$$\text{Max } z = 5x_1 + 3x_2$$

s.t

$$3x_1 + 5x_2 \leq 15$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, x_2 \geq 0.$$

Sol^n: Convert the given LPP into the standard form using slack variables s_1 & s_2

$$\text{Max } Z = 5x_1 + 3x_2 + 0s_1 + 0s_2.$$

S.t.

$$3x_1 + 5x_2 + s_1 = 15$$
$$5x_1 + 2x_2 + s_2 = 10$$
$$x_1, x_2, s_1, s_2 \geq 0$$

C_B	Basic variable	C_j / X_B	5 x_1	3 x_2	0 s_1	0 s_2	Minimum Ratio
0	s_1	15	3	5	1	0	$\frac{15}{3} = 5$
0	s_2	10	$\boxed{5}$ key	2	0	1	$\frac{10}{5} = 2$ ← Outgo
		$Z_j \rightarrow$	0	0	0	0	
$\Delta_j = c_j - z_j$		$\Delta_j \rightarrow$	5	3	0	0	
			↑ incoming vector				
0	s_1	9	0	$\boxed{19/5}$ key	1	$-3/5$	$\frac{45}{19} = 2.3$ ← out
5	x_1	2	1	$2/5$	0	$1/5$	5
		$Z_j \rightarrow$	5	2	0	1	
		$\Delta_j \rightarrow$	0	1	0	-1	
				↑ IN			
3	x_2	$45/19$	0	1	$5/19$	$-3/19$	
5	x_1	$20/19$	1	0	$-2/19$	$5/19$	
		$Z_j \rightarrow$	5	3	$5/19$	$16/19$	
		$\Delta_j \rightarrow$	0	0	$-5/19$	$-16/19$	

Now, all $\Delta_j \le 0$.

So, $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$

$$\therefore \text{Max } z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right)$$

$$= \frac{100}{19} + \frac{135}{19} = \frac{235}{19}$$

2 Solve the following LP using Simplex method:

$$\text{Min } z = x_1 - 3x_2 + 2x_3$$

s.t. $3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

Sol^n: Convert the given LPP into standard form using the slack variables s_1, s_2, s_3.

$$\text{Max } z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$$

s.t.

$$3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0.$$

C_B	Basic Variable	C_j / X_B	-1 x_1	3 x_2	-2 x_3	0 s_1	0 s_2	0 s_3	Mini. Ratio
0	s_1	7	3	-1	3	1	0	0	$\frac{7}{-1} = -$
0	s_2	12	-2	$\boxed{4}$ key	0	0	1	0	$\frac{12}{4} = 3$ ← OUT
0	s_3	10	-4	3	8	0	0	1	$\frac{10}{3} = 3.3$
		z_j	0	0	0	0	0	0	
		Δ_j	-1	3	-2	0	0	0	
				↑ IN					
0	s_1	10	$\boxed{5/2}$ key	0	3	1	$1/4$	0	←
3	x_2	3	$-1/2$	1	0	0	$1/4$	0	-
0	s_3	1	$-5/2$	0	8	0	$-3/4$	1	-
		z_j	$-3/2$	3	0	0	$3/4$	0	-
		Δ_j	$1/2$	0	-2	0	$-3/4$	0	
			↑						
-1	x_1	4	1	0	$6/5$	$2/5$	$1/10$	0	
3	x_2	5	0	1	$3/5$	$1/5$	$3/10$	0	
0	s_3	11	0	0	11	1	$-1/2$	1	
		z_j	-1	3	$3/5$	$1/5$	$4/5$	0	
		Δ_j	0	0	$-13/5$	$-1/5$	$-4/5$	0	

Since, all the values of $\Delta_j \le 0$.

So, we have reached the optimal solution.

$x_1 = 4$ Max $z' = 11$

$x_2 = 5$ Min $z = -11$.

$x_3 = 0$

GRAPHICAL METHOD (used only for 2-variable LPP.

step1 → Consider each inequality-constraint as an eq^n.

2→ Plot each eq^n on the graph.

3→ Shade the common/feasible region. Every point on the line will satisfy the eq^n of the line.

If the inequality-constraint corresponding to that line is '≤', the region below the line lying in 1st quadrant is shaded.

For the inequality-constraint with '≥' sign, the region above the line in 1st quadrant is shaded.

The points lying in the common region will satisfy all the constrains simultaneously.

4→ find the value of Z (objective fn^n) at each corner point of the feasible regio

5→ find out the value of Z (max/min) as per the problem.

Examples:- ① find a geometrical interpretation & sol^n as well for the following LPP:-

Max $Z = 3x_1 + 5x_2$ s.t. $x_1 + 2x_2 \leq 2000$,
$x_1 + x_2 \leq 1500$, $x_2 \leq 600$
& $x_1 \geq 0$, $x_2 \geq 0$

Sol^n step1 consider the inequality as eq^n

$x_1 + 2x_2 = 2000$, $x_1 + x_2 = 1500$, $x_2 = 600$

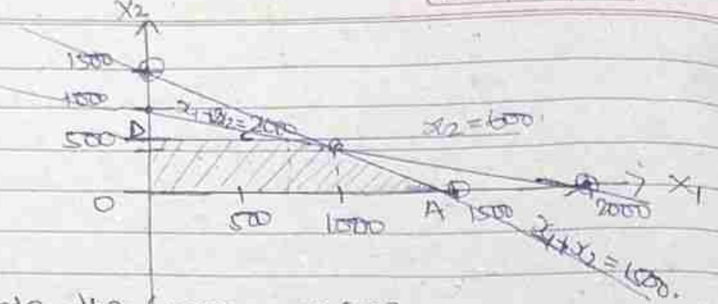
step2 → plot the above eq^ns on graph.

$x_1 + 2x_2 = 2000$

x_1	0	2000
x_2	1000	0

$x_1 + x_2 = 1500$

x_1	0	1500
x_2	1500	0



step3 → Shade the common region.
OABCD is " " "

Step4 → $Z_0=3(0)+5(0)=0$
$Z_A=3(1500)+5(0)=4500$
$Z_B=3(1000)+5(500)=5500$ → Max
$Z_C=3(800)+5(600)=4500$
$Z_D=3(0)+5(600)=$ ~~2500~~ 3000

Step5 ⇒ $x_1=1000$, $x_2=500$ gives required soln.

② Max $Z=8000x_1+7000x_2$
s.t. $3x_1+x_2\le 66$, $x_1+x_2\le 45$, $x_1\le 20$, $x_2\le 40$
& $x_1, x_2 \ge 0$

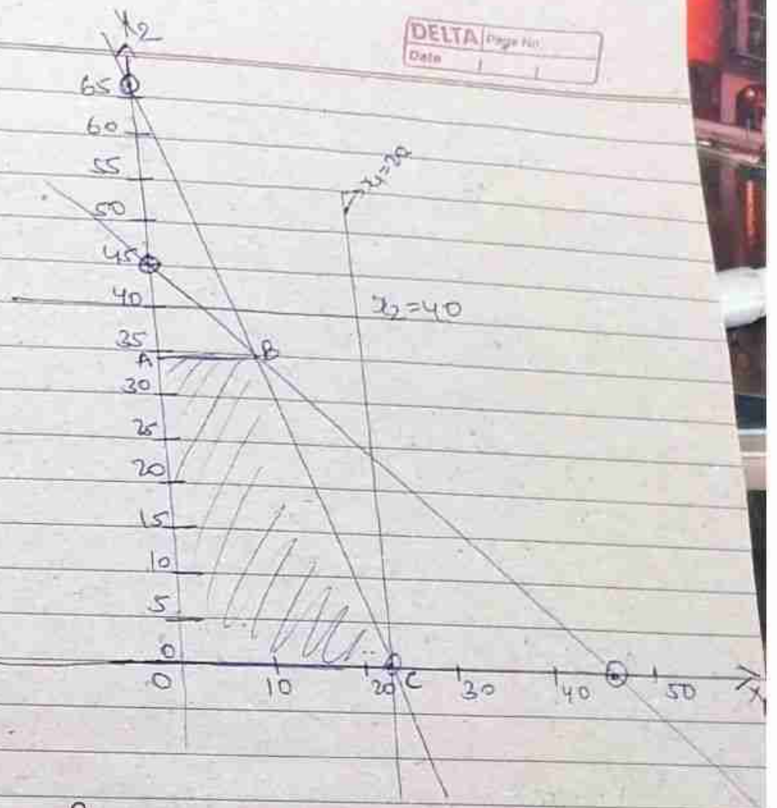
Soln $3x_1+x_2=66$ — ①

x_1	0	22
x_2	66	0

$x_1+x_2=45$ — ⑪

x_1	0	45
x_2	45	0

$Z_0=0$
~~$Z_A=8000$~~ (B is point of intersection of ① & ⑪
∴ solving them, we get coordinates of A.
$3x_1+x_2=66$
$x_1+x_2=45$
$2x_1=21 \Rightarrow x_1=10.5$, $x_2=34.5$
⇒ $Z_B=8000(10.5)+7000(34.5)=\boxed{325000}$ → Max
⇒ $Z_A=8000(0)+7000(34.5)-241,500$
⇒ $Z_C=8000(20)+7000(0)=16000$



③ Minimization Problem

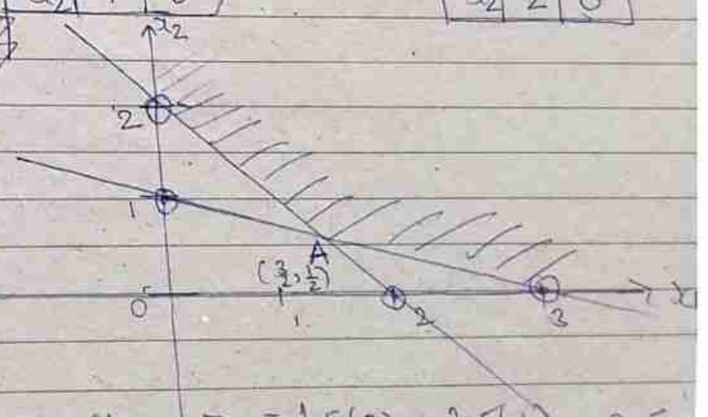
Minimize $Z=1.5x_1+2.5x_2$ s.t. $x_1+3x_2\ge 3$, $x_1+x_2\ge 2$,
$x_1, x_2 \ge 0$

Soln. $x_1+3x_2=3$

x_1	0	3
x_2	1	0

, $x_1+x_2=2$

x_1	0	2
x_2	2	0



Minimum is attained at $Z_A=1.5\left(\frac{3}{2}\right)+2.5\left(\frac{1}{2}\right)=3.5$

④ Problem having Unbounded solⁿ

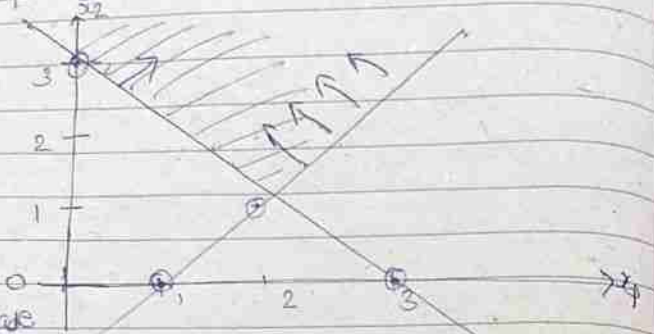
Max $Z = 3x_1 + 2x_2$ s.t. $x_1 - x_2 \le 1$, $x_1 + x_2 \ge 3$ & $x_1, x_2 \ge 0$

solⁿ. $x_1 - x_2 = 1$

x_1	1	2
x_2	0	1

, $x_1 + x_2 = 3$

x_1	0	3
x_2	3	0



Here z can be made arbitrarily large & problem has no finite max value of z. Such problems are said to have unbounded solⁿ.

⑤ Problem with inconsistent system of constraints

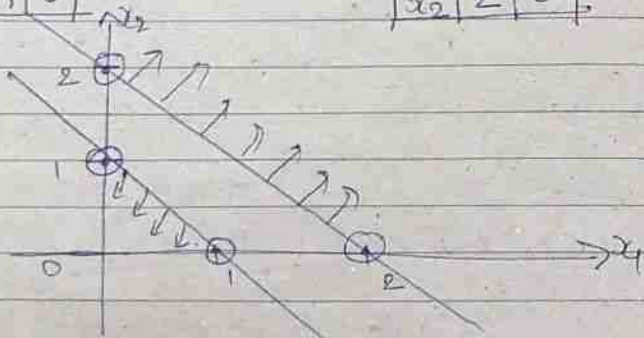
Maximize $z = 3x_1 - 2x_2$, s.t. $x_1 + x_2 \le 1$, $2x_1 + 2x_2 \ge 4$ & $x_1, x_2 \ge 0$.

solⁿ $x_1 + x_2 = 1$

x_1	0	1
x_2	1	0

, $2x_1 + 2x_2 = 4$

x_1	0	2
x_2	2	0



Here there's no point (x_1, x_2) which satisfies both the constraints simultaneously. hence the problem has no solⁿ because the constraints are inconsistent.

⑥ Max $z = 2x_1 + 3x_2$

s.t. $x_1 + x_2 \ge 1$, $5x_1 - x_2 \ge 0$, $x_1 + x_2 \le 6$, $x_1 - 5x_2 \le$

$x_2 - x_1 \ge -1$, $x_2 \le 3$.

solⁿ $x_1 + x_2 = 1$

x_1	0	1
x_2	1	0

, $5x_1 - x_2 = 0$

x_1	0	1
x_2	0	5

$x_1 + x_2 = 6$

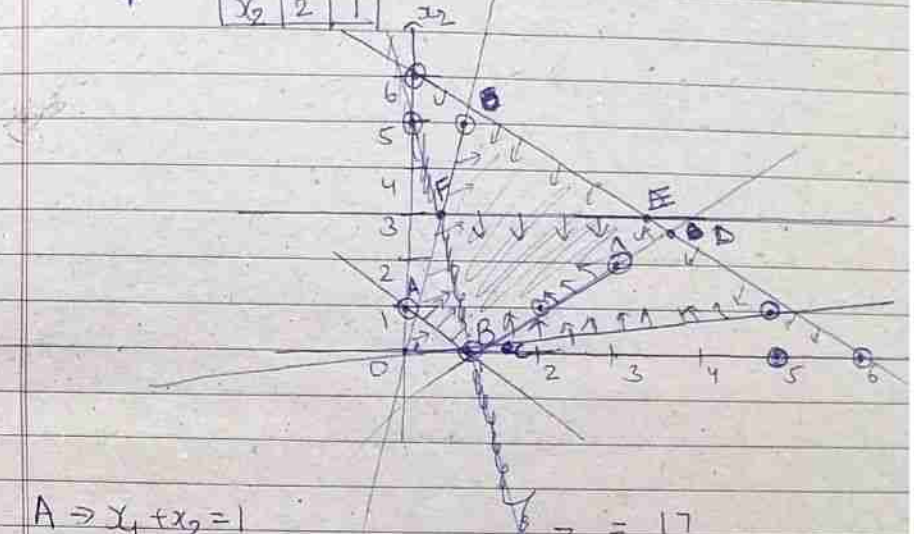
x_1	0	6
x_2	6	0

, $x_1 - 5x_2 = 0$

x_1	5	0
x_2	1	0

$x_2 - x_1 = -1$

x_1	3	2
x_2	2	1



$A \Rightarrow x_1 + x_2 = 1$

$\underline{5x_1 - x_2 = 0}$

$6x_1 = 1 \Rightarrow x_1 = \frac{1}{6}, x_2 = \frac{5}{6}$

$z_A = \frac{17}{6}$

$B \rightarrow x_1 - 5x_2 = 0$

$\underline{x_1 + x_2 = 1}$

$x_2 = \frac{1}{6}, x_1 = \frac{5}{6}$

$z_B = \frac{13}{6}$

$C \rightarrow x_1 - 5x_2 = 0$

$\underline{-x_1 + x_2 = 1}$

$x_2 = \frac{1}{4}, x_1 = \frac{5}{4}$

$z_C = \frac{13}{4}$

$D \rightarrow x_2 - x_1 = -1$

$x_2 + x_1 = 6$

$2x_2 = 5$

$x_2 = \frac{5}{2}, x_1 = \frac{7}{2}$ $\qquad Z_D = \frac{29}{2}$

$E \rightarrow (3,3)$, $Z_E = 15$

$F \rightarrow (\frac{3}{5}, 3)$, $Z_F = \frac{51}{5}$

So, Max at E.

⑦ Problem in which constraints are equations rather than inequalities

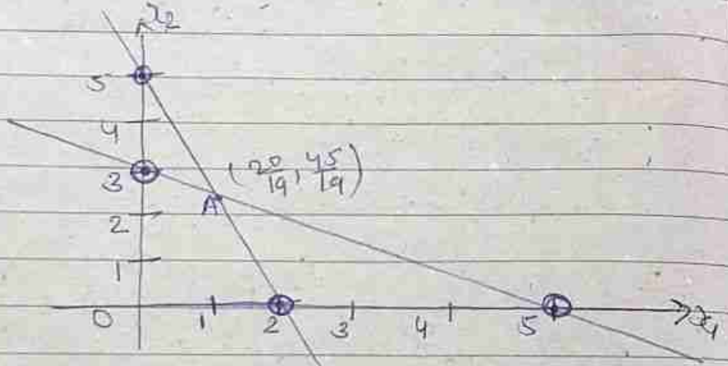
Max $Z = 5x_1 + 3x_2$ s.t. $3x_1 + 5x_2 = 15$, $5x_1 + 2x_2 = 10$, $x_1, x_2 \geq 0$

sol^n $3x_1 + 5x_2 = 15$

x_1	0	5
x_2	3	0

, $5x_1 + 2x_2 = 10$

x_1	0	2
x_2	5	0



since, there is only a single sol^n point $A(\frac{20}{19}, \frac{45}{19})$, there's is nothing to be maximized. //

★ Iso Profit or Iso Cost method

In this method all the previous method steps are used, then plot plot the line $z = 0$ & then draw lines ||el to $z=0$

Q find the sol^n of LPP

Max $z = 45x_1 + 80x_2$ s.t. $5x_1 + 20x_2 \leq 400$

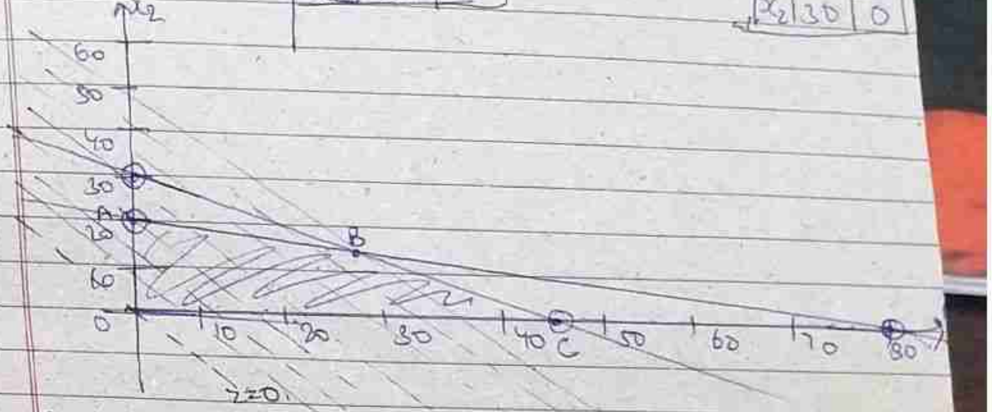
$10x_1 + 15x_2 \leq 450$, $x_1, x_2 \geq 0$

sol^n: $5x_1 + 20x_2 = 400$

x_1	0	80
x_2	20	0

, $10x_1 + 15x_2 = 450$

x_1	0	45
x_2	30	0



B is at intersecting point of eq^{n}s $5x_1 + 20x_2 = 400$ & $10x_1 + 15x_2 = 450$. so, solving these we get $x_1 = 24$, $x_2 = 14$ $\therefore$ $B(14,24) \rightarrow Z = \boxed{2200} \rightarrow$ Max.

$O(0,0) \rightarrow Z = ~~1600~~ 0$

$A(0,20) \rightarrow Z = 1600$

$C(45,0) \rightarrow Z = 2025$

⇒ After ploting ||el lines to $Z=0$, B is the last point which lies on that ||el line. $\therefore$ B is the Max point.

||ly, if we've to get min point then it's the 1st point which lies on the ||el line drawn to $z=0$

✯ General LPP

The general LPP having 'n' decision variables $x_1, x_2, \cdots, x_n$ to maximize / minimize the objective funⁿ, $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ subject to the 'm' constraints

$$a_{11}x_1 + a_{12}x_2 + - - - + a_{1n}x_n \ (\leq / \geq / =)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + - - - + a_{2n}x_n \ (\leq / \geq / =)\ b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + - - - + a_{mn}x_n \ (\leq / \geq / =)\ b_m$$

& non-ve restrictions, $x_1, x_2, \cdots, x_n \geq 0$.

✯ SLACK VARIABLE.

If a constraint has ≤ sign, then in order to make it an equality, we've to add something +ve to the L.H.S.

→ The non-ve variable which is added to the L.H.S. of the constraint to convert it into eqⁿ is called slack variable.

Ex:- $x_1 + x_2 \leq 2$

To convert it into equality we add s_1, to it

i.e. $x_1 + x_2 + s_1 = 2$ → slack variable.

✯ SURPLUS VARIABLE

If a constraint has ≥ sign, then in order to make it an equality we've to subtract something non -ve from its L.H.S.

→ Thus the +ve variable which is subtracted from L.H.S. of the constraint to convert it into eqⁿ is called surplus variable

Ex:- $2x_1 + 3x_2 \geq 7$

To convert it into equality we've to subtract s_2 from it.

i.e. $x_1 + x_2 - s_2 = 7$ → surplus variable.

Std. form of LPP

Step1 All the constraints should be converted into the eqⁿs except for the non-vity restrictions which remain as inequalities (≥ 0).

Step2 The R.H.S. of each constraint should be made non-ve (if not)

Step3 All variables must have non-ve values. A variable which is unrestricted in sign is equivalent to the difference betⁿ two non-ve variables.

Step4 The objective funⁿ should be of maximization form.

Some Imp defⁿs

① **Solⁿ to LPP** :- Any set $x = \{x_1, x_2, \cdots, x_{n+m}\}$ is called solⁿ of LPP if it satisfies the set of constraints.

② **Feasible Solⁿ** → Any set $x = \{x_1, x_2, \cdots, x_{n+m}\}$ of variables is called feasable solⁿ of LPP if it satisfies the set of constraints & non-ve restrictions also.

③ **Basic Solⁿ** → A basic solⁿ to the set of constraints is a solⁿ obtained by setting any 'n' variables among (n+m) variables equal to zero & solving for the remaining 'm' variables provided the determinant of the coefficients of these 'm'

variables is non-zero.
Such 'm' variables are called Basic Variables & remaining 'n' zero valued variables are called non basic variables.

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④ Basic feasible solⁿ

A basic feasible solⁿ is a basic solⁿ which also satisfies the non-negativity restriction.

→ These are of two types.

a) Non-degenerate BFS –
All the m basic variables are +ve & remaining 'n' variables will be zero

b) Degenerate BFS –
A BFS is called degenerate, if one or more basic variables are zero-valued.

⑤★ Optimum BFS
A BFS is s.t.b. optimum, if it also optimizes (maximizes or minimizes) the objective funⁿ.

⑥ Unbounded Solⁿ
If the value of the objective funⁿ Z can be ↑sed or ↓sed indefinitely, such solⁿs are called Unbounded solⁿs.

Q Convert the given LPP into std. form

Min $Z = 2x_1 - 3x_2$

s.t. $2x_1 + x_2 \geq -4$

$x_1 + x_2 \leq 7$

$x_1, x_2 \geq 0$

Solⁿ. To convert the given LPP into std form use the slack variables

Max $Z' = -2x_1 - 3x_2$ where $Z' = -Z$

s.t. $-2x_1 - x_2 \leq 4$

$x_1 + x_2 \leq 7$

$-2x_1 - x_2 + s_1 = 4$

$x_1 + x_2 + s_2 = 7$

$x_1, x_2, s_1, s_2 \geq 0$

★ SIMPLEX METHOD

Max $Z = 5x_1 + 3x_2$

s.t. $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

Solⁿ Convert the given L.P.P. into the std. form using slack variables s_1 & s_2

Max $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

s.t. $3x_1 + 5x_2 + s_1 = 15$

known as decision variables. $5x_1 + 2x_2 + s_2 = 10$

$x_1, x_2, s_1, s_2 \geq 0$

C_B: Coeff. of basic variable in objective funⁿ

X_B: Basic feasible solⁿ

C_j: Coeff of variables in objective funⁿ.

Δ_j: Net evaluation which is computed using formula

$$\Delta_j = Z_j - C_j$$

$Z = C_B X_B$

$Z_j = \cancel{C_j X_j}\ C_B X_j$

X_k: Key Column → wo column jisme Δ_j ke corresponding highest +ve variable hoga.

Key Row → wo row jo Min. ratio column me min. value degi.

Key element → the point where key column & key row intersect.

Basic variable wo variable hote h jo matrix me unit matrix bnaate h.

		$C_j \to$	35	3	0	0	
C_B	Basic Variables	X_B	x_1	x_2	s_1	s_2	Minimum ratio X_B/X_K for $X_K>0$
0	s_1	15	3	5	1	0	$X_B/X_1 = \frac{15}{3} = 5$
0	s_2	10	$\boxed{5}$ key element	2	0	1	$X_B/X_1 = \frac{10}{5} = 2$ ← outgoing so keyrow is 2nd one
$Z = C_B X_B = 0$		$Z_j \to$	0	0	0	0	
	$\Delta_j = C_j - Z_j \to$		⑤	3	0	0.	
			↑ incoming vector so Key Column is x_1				
0	s_1	9	0	$\boxed{19/5}$	1	$-3/5$	$X_B/X_2 = 9 \div \frac{19}{5} = \frac{45}{19}$ ← out going
5	x_1	2	1	$2/5$	0	$1/5$	$X_B/X_2 = 2 \div \frac{2}{5} = 5$
$Z = C_B X_B = 10$		$Z_j \to$	5	2	0	1	~~$X_B/X_2 =$~~
	$\Delta_j = C_j - Z_j$		0	①	0	-1	
				↑ incoming vector so Key Column is x_2			
3	x_2	$45/19$	0	1	$5/19$	$\frac{-3}{19}$	
5	x_1	$20/19$	1	0	$-2/19$	$5/19$	
		$Z_j \to$	5	3	$\frac{5}{19}$	$\frac{16}{19}$	
		$\Delta_j \to$	0	0	$\frac{-5}{19}$	$\frac{-16}{19}$	

Now all $\Delta_j \le 0$

so, $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$

Now, $\text{Max } Z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$

Q2 $\text{Max } Z = 5x_1 + 7x_2$ s.t. $x_1 + x_2 \le 4$, $3x_1 - 8x_2 \le 24$, $10x_1 + 7x_2 \le 35$

Sol $\text{Max } Z = 5x_1 + 7x_2 + 0s_1 + 0s_2 + 0s_3$

s.t. $x_1 + x_2 + s_1 = 4$, $3x_1 - 8x_2 + s_2 = 24$, $10x_1 + 7x_2 + s_3 = 3$

		$C_j \to$	5	7	0	0	0	
C_B	Basic Variables	X_B	x_1	x_2	s_1	s_2	s_3	Minimum ratio X_B/X_K
0	s_1	4	1	$\boxed{1}$	1	0	0	$X_B/X_2 = \frac{4}{1} = 4$ →
0	s_2	24	3	-8	0	1	0	$= \frac{24}{-8}$
0	s_3	35	10	7	0	0	1	$= \frac{35}{7} = 5$
		$Z_j \to$	0	0	0	0	0	
		$\Delta_j \to$	5	⑦	0	0	0	
				↑				
7	x_2	4	1	1	1	0	0	
0	s_2	56	11	0	8	1	0	
0	s_3	7	3	0	-7	0	1	
		$Z_j \to$	7	7	7	0	0	
		$\Delta_j \to$	-2	0	-7	0	0	

since all $C_j - Z_j \le 0$

so, $x_1 = 0$,

$x_2 = 4$

$\text{Max } Z = 28$

Q3 $\text{Max } Z = x_1 + x_2 + 3x_3$

s.t. $3x_1 + 2x_2 + x_3 \le 3$

$2x_1 + x_2 + 2x_3 \le 2$

$x_1, x_2, x_3 \ge 0$.

Sol $\text{Max } Z = x_1 + x_2 + 3x_3 + 0s_1 + 0s_2$

s.t. $3x_1 + 2x_2 + x_3 + s_1 = 3$

$2x_1 + x_2 + 2x_3 + s_2 = 2$.

C_B	Basic Variable	x_B	x_1	x_2	x_3	s_1	s_2	Minimum ratio x_B/x_k
		$C_j \to$	1	1	3	0	0	
0	s_1	3	3	2	1	1	0	$x_B/x_3 = \frac{3}{1} = 3$
0	s_2	2	2	1	$\boxed{2}$	0	1	$= \frac{2}{2} = 1 \rightarrow$
		$Z_j \to$	0	0	0	0	0	
		$\Delta_j \to$	1	1	③	0	0	
					↑			
0	s_1	2	2	$\frac{3}{2}$	0	1	$-\frac{1}{2}$	
3	x_3	1	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	
		$Z_j \to$	3	$3/2$	3	0	$3/2$	
		$\Delta_j \to$	-2	$-1/2$	0	0	$-1/2$	

since $\Delta_j \leq 0$

hence, $x_1 = 0,\ x_2 = 0,\ x_3 = 1$

$\therefore$ Max $Z = 3$ //

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Q Solve the following LPP using simplex method.

Min. $Z = x_1 - 3x_2 + 2x_3$

s.t. $3x_1 - x_2 + 3x_3 \leq 7$, $-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$.

Sol Convert the given LPP into the std. form using slack variables s_1, s_2, s_3

Max $z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$ $(z' = -z)$

s.t. $3x_1 - x_2 + 3x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

C_B	Basic Variable	x_B	x_1	x_2	x_3	s_1	s_2	s_3	Min Ratio.
		$C_j \to$	-1	3	-2	0	0	0	
0	s_1	7	3	-1	3	1	0	0	$x_B/x_2 = \frac{7}{-1} = -7$
0	s_2	12	-2	$\boxed{4}$	0	0	1	0	$\frac{12}{4} = 3 \rightarrow$
0	s_3	10	-4	3	8	0	0	1	$\frac{10}{3} = 3.33$
		$Z_j \to$	0	0	0	0	0	0	
		$\Delta_j \to$	-1	③	-2	0	0	0	
				↑					
0	s_1	10	$\boxed{\frac{5}{2}}$	0	3	1	$\frac{1}{4}$	0	$x_B/x_1 = 10 \div \frac{5}{2} = 4 \rightarrow$
3	x_2	3	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	$3 \div (-\frac{1}{2}) = -6$
0	s_3	1	$-5/2$	0	8	0	$-3/4$	1	$1 \div (-\frac{5}{2}) = -$
		$Z_j \to$	$-\frac{3}{2}$	3	0	0	$\frac{3}{4}$	0	
		$\Delta_j \to$	$\textcircled{\frac{1}{2}}$	0	-2	0	$-\frac{3}{4}$	0	
			↑						
-1	x_1	4	1	0	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	
3	x_2	5	0	1	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	0	
0	s_3	11	0	0	11	1	$-\frac{1}{2}$	1	
		$Z_j \to$	-1	3	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{8}{10}$	0	
		$\Delta_j \to$	0	0	$\frac{-13}{5}$	$-\frac{1}{5}$	$\frac{-8}{10}$	0	

$x_1 = 4,\ x_2 = 5,\ x_3 = 0$

Max $Z' = -4 + 15 = 11$

Min $Z = -11$ //

Assignment

Q1) ~~Min $z = x_1 - 3x_2 + 2x_3$ s/t. $3x_1 - x_2 + 2x_3 \le 7$, $-2x_1 + 4x_2 \le 12$~~ Max. $Z = 3x_1 + 2x_2 + 5x_3$ s.t.

$x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$, $x_1 + 4x_2 \le 42$

and $x_1, x_2, x_3 \ge 0$.

Sol Std. form → Max $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

s.t. $x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 4x_2 + s_3 = 420$.

$c_j \rightarrow$ 3 2 5 0 0 0

C_B	Basic Variables	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min. Ratio
0	s_1	430	1	2	1	1	0	0	$X_B/x_3 = 430$
0	s_2	460	3	0	$\boxed{2}$	0	1	0	$= \frac{460}{2}$ →
0	s_3	420	1	4	0	0	0	1	—
		$Z_j \rightarrow$	0	0	0	0	0	0	
		$\Delta_j \rightarrow$	3	2	⑤	0	0	0	
					↑				
0	s_1	200	$-1/2$	$\boxed{2}$	0	1	$-1/2$	0	$X_B/x_2 = 100$ →
5	x_3	230	$3/2$	0	1	0	$1/2$	0	—
0	s_3	420	1	4	0	0	0	1	105
		$Z_j \rightarrow$	$15/2$	0	5	0	$5/2$	0	
		$\Delta_j \rightarrow$	$-9/2$	②	0	0	$-5/2$	0	
				↑					
2	x_2	100	$-1/4$	1	0	$1/2$	$-1/4$	0	
5	x_3	230	$3/2$	0	1	0	$1/2$	0	
0	s_3	20	2	0	0	-2	1	1	
		$Z_j \rightarrow$	7	2	5	1	2	0	
		$\Delta_j \rightarrow$	-4	0	0	-1	-2	0	

So, $x_1 = 0$, $x_2 = 100$, $x_3 = 230$

max $Z = 1350$ //

Q2 Max $Z = 3x_1 + 5x_2 + 4x_3$ s.t. $2x_1 + 3x_2 \le 8$, $2x_2 + 5x_3 \le 10$, $3x_1 + 2x_2 + 4x_3 \le 15$ and $x_1, x_2, x_3 \ge 0$

Sol Std. form → Max $Z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

s.t. $2x_1 + 3x_2 + s_1 = 8$

$2x_2 + 5x_3 + s_2 = 10$

$3x_1 + 2x_2 + 4x_3 + s_3 = 15$

$c_j \rightarrow$ 3 5 4 0 0 0

C_B	Basic variable	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min Ratio
0	s_1	8	2	$\boxed{3}$	0	1	0	0	$X_B/x_2 = \frac{8}{3}$ →
0	s_2	10	0	2	5	0	1	0	5
0	s_3	15	3	2	4	0	0	1	$\frac{15}{2}$
		$Z_j \rightarrow$	0	0	0	0	0	0	
		$\Delta_j \rightarrow$	3	⑤	4	0	0	0	
				↑					
5	x_2	$8/3$	$2/3$	1	0	$1/3$	0	0	$X_B/x_3 =$ —
0	s_2	$14/3$	$-4/3$	0	$\boxed{5}$	$-2/3$	1	0	$\frac{14}{3} \div 5 = \frac{14}{15}$ →
0	s_3	$29/3$	$5/3$	0	4	$-2/3$	0	1	$\frac{29}{3} \div 4 = \frac{29}{12}$
		$Z_j \rightarrow$	$10/3$	5	0	$5/3$	0	0	
		$\Delta_j \rightarrow$	$1/3$	0	④	$-5/3$	0	0	
					↑				
5	x_2	$8/3$	$2/3$	1	0	$1/3$	0	0	$X_B/x_1 = 4$
4	x_3	$14/15$	$-4/15$	0	1	$-2/15$	$\frac{1}{5}$	0	—
0	s_3	$89/15$	$\boxed{41/15}$	0	0	$-2/15$	$-4/15$	1	$\frac{89}{41}$ →
		$Z_j \rightarrow$	$34/15$	5	4	$17/15$	$\frac{4}{5}$	0	
		$\Delta_j \rightarrow$	($11/15$)	0	0	$-17/15$	$-\frac{4}{5}$	0	
			↑						
5	x_2	$50/41$	0	1	0	$\frac{15}{41}$	$\frac{8}{41}$	$\frac{-10}{41}$	
4	x_3	$62/41$	0	0	1	$\frac{-6}{41}$	$\frac{5}{41}$	$\frac{4}{41}$	
3	x_1	$89/41$	1	0	0	$\frac{-2}{41}$	$\frac{-12}{41}$	$\frac{15}{41}$	
		$Z_j \rightarrow$	3	5	4	$\frac{45}{41}$	$\frac{24}{41}$	$\frac{11}{41}$	
		$\Delta_j \rightarrow$	0	0	0	$-\frac{45}{41}$	$-\frac{24}{41}$	$-\frac{11}{41}$	

$x_1 = \frac{89}{41} \cdot x_2 = \frac{50}{41}, x_3 = \frac{62}{41}$, $\max z = \frac{765}{41}$.

Q3 Minimize $z = x_1 - 3x_2 + 2x_3$ s.t. $3x_1 - x_2 + 2x_3 \le 7$, $-2x_1 + 4x_2 \le 12$, $-4x_1 + 3x_2 + 8x_3 \le 10$ & $x_1, x_2, x_3 \ge 0$

Sol. Std. form → $\max z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

s.t. $3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$c_j \to$ -1 3 -2 0 0 0

C_B	Basic Variable	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min. Ratio
0	s_1	7	3	-1	2	1	0	0	x_B/x_2 = –
0	s_2	12	-2	$\boxed{4}$	0	0	1	0	3 →
0	s_3	10	-4	3	8	0	0	1	3.333
		$Z_j \to$	0	0	0	0	0	0	
		$\Delta_j \to$	-1	3 ↑	-2	0	0	0	
0	s_1	10	$\boxed{5/2}$	0	2	1	-1/4	0	x_B/x_1 = 4 →
3	x_2	3	-1/2	1	0	0	$\frac{1}{4}$	0	–
0	s_3	1	-5/2	0	8	0	-3/4	1	–
		$Z_j \to$	-3/2	3	0	0	3/4	0	
		$\Delta_j \to$	1/2 ↑	0	-2	0	-3/4	0	
-1	x_1	4	1	0	4/5	$\frac{2}{5}$	$\frac{1}{10}$	0	
3	x_2	5	0	1	2/5	$\frac{1}{5}$	$\frac{3}{10}$	0	
0	s_3	11	0	0	10	1	-1/2	1	
		$Z_j \to$	-1	3	2/5	1/5	$\frac{4}{5}$	0	
		$\Delta_j \to$	0	0	$-\frac{12}{5}$	-1/5	$-\frac{4}{5}$	0	

$x_1 = 4, x_2 = 5, x_3 = 0$

$\max z' = 11$ $\min z = -11$

Q4 $\max z = 3x_1 + 2x_2$ s.t. $2x_1 + x_2 \le 5$, $x_1 + x_2 \le 3$, $x_1, x_2 \ge 0$

Sol Std. form: $\max z = 3x_1 + 2x_2 + 0s_1 + 0s_2$

s.t. $2x_1 + x_2 + s_1 = 5$, $x_1 + x_2 + s_2 = 3$

$c_j \to$ 3 2 0 0

C_B	Basic Variables	X_B	x_1	x_2	s_1	s_2	Min. ratio
0	s_1	5	$\boxed{2}$	1	1	0	$x_B/x_1 = \frac{5}{2}$ = 2.5 →
0	s_2	3	1	1	0	1	= 3
		$Z_j \to$	0	0	0	0	
		$\Delta_j \to$	3 ↑	2	0	0	
3	x_1	5/2	1	1/2	1/2	0	x_B/x_2 = 5
0	s_1	1/2	0	$\boxed{1/2}$	-1/2	1	= 1 →
		$Z_j \to$	3	3/2	3/2	0	
		$\Delta_j \to$	0	1/2 ↑	-3/2	0	
3	x_1	2	1	0	1	-1	
2	x_2	1	0	1	-1	2	
		$Z_j \to$	3	2	1	1	
		$\Delta_j \to$	0	0	-1	-1	

$\Rightarrow x_1 = 2, x_2 = 1$ so, $\max z = 8$

Q5 $\max z = 5x_1 + 3x_2$ s.t. $x_1 + x_2 \le 2$, $5x_1 + 2x_2 \le 10$, $3x_1 + 8x_2 \le 12$, $x_1, x_2 \ge 0$

Sol Std. form → $\max z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$

s.t. $x_1 + x_2 + s_1 = 2$

$5x_1 + 2x_2 + s_2 = 10$

$3x_1 + 8x_2 + s_3 = 12$

		$c_j \to$	5	3	0	0	0	
C_B	Basic Variable	X_B	x_1	x_2	s_1	s_2	s_3	Min Ratio
0	s_1	2	$\boxed{1}$	1	1	0	0	$x_B/x_1 = 2 \rightarrow$
0	s_2	10	$\boxed{5}$	$\boxed{2}$	0	1	0	2
0	s_3	12	3	0	0	0	1	4
		$Z_j \to$	0	0	0	0	0	
		$\Delta_j \to$	$\textcircled{5}$ ↑	3	0	0	0	
5	x_1	2	1	1	1	0	0	
0	s_2	0	0	-3	-5	1	0	
0	s_3	6	0	5	-3	0	1	
		$Z_j \to$	5	5	5	0	0	
		$\Delta_j \to$	0	-2	-5	0	0	

$x_1 = 2,\ x_2 = 0$ so, Max $Z = 10$ //

Q6 Max $Z = 2x_1 + 4x_2 + x_3 + x_4$ s.t. $2x_1 + x_2 + 2x_3 + 3x_4 \le 12$
$3x_1 + 2x_3 + 2x_4 \le 20$, $2x_1 + x_2 + 4x_3 \le 16$,
$x_1, x_2, x_3, x_4 \ge 0$

Sol Std. form, Max $Z = 2x_1 + 4x_2 + x_3 + x_4 + 0s_1 + 0s_2 + 0s_3$

s.t. $2x_1 + x_2 + 2x_3 + 3x_4 + s_1 = 12$
$3x_1 + 2x_3 + 2x_4 + s_2 = 20$
$2x_1 + x_2 + 4x_3 + s_3 = 16$

		$c_j \to$	2	4	1	1	0	0	0	
C_B	Basic Variable	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	Min Ratio
0	s_1	12	2	$\boxed{1}$	2	3	1	0	0	$x_B/x_2 = 12 \rightarrow$
0	s_2	20	3	0	2	2	0	1	0	—
0	s_3	16	2	1	4	0	0	0	1	16
		$Z_j \to$	0	0	0	0	0	0	0	
		$\Delta_j \to$	2	$\textcircled{4}$ ↑	1	1	0	0	0	
4	x_2	12	2	1	2	3	1	0	0	
0	s_2	20	3	0	2	2	0	1	0	
0	s_3	4	0	0	2	-3	-1	0	1	
		$Z_j \to$	8	4	8	12	4	0	0	
		$\Delta_j \to$	-6	0	-7	-11	-4	0	0	

So, $x_1 = 0,\ x_2 = 12,\ x_3 = 0$
Max $Z = 48$ //

Q7 Max $Z = 2x + 5y$ s.t. $x + y \le 600$; $0 \le x \le 400$,
$0 \le y \le 300$.

Sol Std. form: Max $Z = 2x + 5y + s_1 + s_2 + s_3$
s.t. $x + y + s_1 = 600$
$x + s_2 = 400$, $y + s_3 = 300$

		$c_j \to$	2	5	0	0	0	
C_B	Basic variable	X_B	x	y	s_1	s_2	s_3	Min Ratio
0	s_1	600	1	1	1	0	0	$x_B/x_2 = 600$
0	s_2	400	1	0	0	1	0	—
0	s_3	300	0	$\boxed{1}$	0	0	1	$300 \rightarrow$
		$Z_j \to$	0	0	0	0	0	
		$\Delta_j \to$	2	$\textcircled{5}$ ↑	0	0	0	
0	s_1	300	$\boxed{1}$	0	1	0	0	$x_B/x_1 = 300 \rightarrow$
0	s_2	400	1	0	0	1	0	= 400
5	y	300	0	1	0	0	1	= —
		$Z_j \to$	0	5	0	0	0	
		$\Delta_j \to$	$\textcircled{2}$ ↑	0	0	0	0	
2	x	300	1	0	1	0	0	
0	s_2	100	0	0	-1	1	0	
5	y	300	0	1	0	0	1	
		$Z_j \to$	2	5	2	0	5	
		$\Delta_j \to$	0	0	-2	0	-5	

→ $x = 200$; $y = 300$

→ $\max z = 2100$

8/Aug/23

PROBLEM OF DEGENERACY

Sometimes the minimum ratio may not be unique. This causes the problem of degeneracy.

1. Tie in outgoing For this type, take the unit matrix & divide by corresponding no.

Q. $\text{Max } Z = 3x_1 + 9x_2$ s.t. $x_1 + 4x_2 \le 8$, $x_1 + 2x_2 \le 4$, $x_1, x_2 \ge 0$

Sol. Std. form ⇒ $\text{Max } Z = 3x_1 + 9x_2 + 0s_1 + 0s_2$

s.t. $x_1 + 4x_2 + s_1 = 8$ and $x_1 + 2x_2 + s_2 = 4$

		$C_j \to$	3	9	0	0	
C_B	Basic Variables	X_B	x_1	x_2	s_1	s_2	Min Ratio
0	s_1	8	1	4	1	0	$X_B/x_2 = \frac{8}{4} = 2$
0	s_2	4	1	$\boxed{2}$	0	1	2 →
		$Z_j \to$	0	0	0	0	
		$\Delta_j \to$	3	⑨ ↑	0	0	

	s_1	s_2
1st	1/4	0
	0/2	1

min so row 2 is outgoing.

0	s_1	0	-1	0	1	-2
9	x_2	2	1/2	1	0	1/2
		$Z_j \to$	9/2	9	0	9/2
		Δ_j	-3/2	0	0	-9/2

So, $x_1 = 0$, $x_2 = 2$, $\text{Max } z = 18$

2. Tie in Incoming variable –

It occurs when most +ve. value of Δ_j is not unique.

Case 1 → When tie occurs betn decision variables, then arbitrarily select any " ".

Case 2 → If there's a tie betn slack or surplus variable then, arbitrarily select any slack or surplus variable.

Case 3 → If there's a tie betn a decision variable & slack/surplus variable, then always give priority to decision variable.

Unbounded solⁿ

$\text{Max } Z = 107x_1 + x_2 + 2x_3$ s.t. $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + (\frac{1}{2})x_2 - 6x_3 \le 5$, $3x_1 - x_2 - x_3 \le 0$,

$x_1, x_2, x_3 \ge 0$

Sol. Std. form → $\text{Max } Z = 107x_1 + x_2 + 2x_3 + 0x_4 + 0s_1 + 0s_2$

s.t. $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + \frac{1}{2}x_2 - 6x_3 + s_1 = 5$

$3x_1 - x_2 - x_3 + s_2 = 0$

		$C_j \to$	107	1	2	0	0	0	
C_B	Basic Variables	X_B	x_1	x_2	x_3	x_4	s_1	s_2	Min Ratio
0	x_4	7	14	1	-6	3	0	0	$X_B/x_1 = \frac{7}{14}$
0	s_1	5	$\boxed{16}$	1/2	-6	0	1	0	$\frac{5}{16}$ →
0	s_2	0	3	-1	-1	0	0	1	—
		$Z_j \to$	0	0	0	0	0	0	
		$\Delta_j \to$	(107) ↑	1	2	0	0	0	
0	x_4	21/8	0	9/16	-3/4	3	-7/8	0	
107	x_1	5/16	1	1/32	$-\frac{3}{8}$	0	1/16	0	
0	s_2	$\frac{-15}{16}$	0	$-\frac{35}{32}$	-17/8	0	$\frac{-3}{16}$	1	

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$Z=\frac{535}{16}$ $Z_j\rightarrow$	107	$\frac{107}{32}$	$-\frac{37}{8}$	0	$\frac{107}{16}$	0
$\Delta_j\rightarrow$	0	$-\frac{75}{32}$	$\frac{337}{8}$	0	$-\frac{107}{16}$	0

So, sol^n is unbounded

~~★ ALTERNATE OPTIMUM SOL^n~~

★ **Big - M - Method / CHARNE'S PENALTY METHOD**

→ $3x_1+2x_2\geq 1$ → surplus var. → artificial var.

$3x_1+2x_2-s_1+a_1=1$

→ $x_1+x_2=1$ → artificial var.

$x_1+x_2+a_2=1$

→ $2x_1+3x_2\leq 1$

$2x_1+3x_2+s_1=1$ → slack

Note → Once artificial variable exist it can't enter again then, stop calculating for that var.

Q Solve the foll. LPP using Big-M Method

Max $Z=-2x_1-x_2$

s.t. $3x_1+x_2=3$, $4x_1+3x_2\geq 6$, $x_1+2x_2\leq 4$, $x_1, x_2\geq 0$

Sol^n Std. form → Max $Z=-2x_1-x_2+0s_1+0s_2-Ma_1-Ma_2$ (very ~~Max~~ large coeff. of artificial var.)

s.t. $3x_1+x_2+a_1=3$

$4x_1+3x_2-s_1+a_2=6$

$x_1+2x_2+s_2=4$

		$C_j\rightarrow$	-2	-1	0	0	$-M$	$-M$	
C_B	Basic var	X_B	x_1	x_2	s_1	s_2	a_1	a_2	Min Ratio
$-M$	a_1	3	$\boxed{3}$	1	0	0	1	0	$X_B/x_1=1$ →
$-M$	a_2	6	4	3	-1	0	0	1	$\frac{6}{4}$
0	s_2	4	1	2	0	1	0	0	4
$Z=C_BX_B$		$Z_j\rightarrow$	$-7M$	$-4M$	M	0	$-M$	$-M$	
$=-9M$		$\Delta_j\rightarrow$	$7M-2$ ↑	$4M-1$	$-M$	0	0	0	

5: Select the smallest Quantity amongst the cells marked with

the closed

to

→ std. eq^n $Z=3x_1+2x_2+0s_1+0s_2-M.a_1$

s.t. $2x_1+x_2+s_1=2$

$3x_1+4x_2-s_2+a_1=12$

-2	x_1	1	1	$\frac{1}{3}$	0	0	$\cancel{\frac{1}{3}}$	0	3
$-M$	a_2	2	0	$\boxed{\frac{5}{3}}$	-1	0	X	1	$\frac{6}{5}$ →
0	s_2	3	0	$\frac{5}{3}$	0	1	X	0	$\frac{9}{5}$
$Z=-2-2M$		$Z_j\rightarrow$	-2	$-\frac{2}{3}-\frac{5}{3}M$	M	0	X	$-M$	
		$\Delta_j\rightarrow$	0	$-\frac{1}{3}+\frac{5}{3}M$ ↑	$-M$	0	X	0	
-2	x_1	$\frac{3}{5}$	1	0	$\frac{1}{5}$	0	X	X	
-1	x_2	$\frac{6}{5}$	0	1	$-\frac{3}{5}$	0	X	X	
0	s_2	1	0	0	1	1	X	X	
$Z=-\frac{12}{5}$		$Z_j\rightarrow$	-2	-1	5	0	X	X	
		$\Delta_j\rightarrow$	0	0	$-\frac{1}{5}$	0	X	X	

$x_1=\frac{3}{5}$, $x_2=\frac{6}{5}$, $Z=-\frac{12}{5}$ //.

Q Max $Z=3x_1+2x_2$ s.t. $2x_1+x_2\leq 2$, $3x_1+4x_2\geq 12$

$x_1, x_2\geq 0$

		$C_j\rightarrow$	3	2	0	0	$-M$	
C_B	Basic var	X_B	x_1	x_2	s_1	s_2	a_1	Min Ratio
0	s_1	2	2	$\boxed{1}$	1	0	0	2 →
$-M$	a_1	12	3	4	0	-1	1	3
$Z=-12M$		$Z_j\rightarrow$	$-3M$	$-4M$	0	M	$-M$	
		$\Delta_j\rightarrow$	$3+3M$	$2+4M$ ↑	0	$-M$	0	
2	x_2	2	2	1	1	0	0	
$-M$	a_1	4	-5	0	-4	-1	1	
$Z=4-4M$		$Z_j\rightarrow$	$4+5M$	2	$2+4M$	M	$-M$	
		$\Delta_j\rightarrow$	$-1+5M$	0	$-2-4M$	$-M$	0	

Here all Δj are -ve or 0. So acc. to the optimality cond^n this sol^n is optimal. such sol^n may be called pseudo-optimal sol^n.
It means it doesn't satisfy all the constraints but it satisfies the optimality cond^n of the simplex method.

① Max. $Z = x_1 + 2x_2 + 3x_3 - x_4$ s.t. $x_1 + 2x_2 + 3x_3 = 15$, $2x_1 + x_2 + 5x_3 = 20$, $x_1 + 2x_2 + x_3 + x_4 = 10$, $x_1, x_2, x_3, x_4 \geq$
Using Big-M-method.

Sol^n std. form, Max. $Z = x_1 + 2x_2 + 3x_3 - x_4 - Ma_1 - Ma_2$

s.t. $x_1 + 2x_2 + 3x_3 + a_1 = 15$

$2x_1 + x_2 + 5x_3 + a_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

	$c_j \rightarrow$		1	2	3	-1	-M	-M	
C_B	Basic Var.	x_B	x_1	x_2	x_3	x_4	a_1	a_2	Min Ratio
-M	a_1	15	1	2	3	0	1	0	$\frac{15}{3} = 5$
-M	a_2	20	2	1	$\boxed{5}$	0	0	1	$\frac{20}{5} = 4 \rightarrow$
-1	x_4	10	1	2	1	1	0	0	10
$Z = -35M - 10$		$Z_j \rightarrow$	-3M-1	-3M-2	-8M-1	-1	-M	-M	
		$\Delta_j \rightarrow$	3M+2	3M+4	$\underset{\uparrow}{(8M+4)}$	0	0	0	
-M	a_1	3	-1/5	$\boxed{7/5}$	0	0	1	X	$15/7 \rightarrow$
3	x_3	4	2/5	1/5	1	0	0	X	20
-1	x_4	6	3/5	9/5	0	1	0	X	30/9
$Z = -3M + 6$		$Z_j \rightarrow$	$\frac{(M+3)}{5}$	$\frac{(-7M-6)}{5}$	3	-1	-M	X	
		$\Delta_j \rightarrow$	$\frac{2-M}{5}$	$\underset{\uparrow}{\left(\frac{7M+16}{5}\right)}$	0	0	0	X	
2	x_2	15/7	-1/7	1	0	0	X	X	~~15~~ —
3	x_3	25/7	3/7	0	1	0	X	X	25/3
-1	x_4	15/7	$\boxed{6/7}$	0	0	1	X	X	$\frac{15}{6} \rightarrow$
$Z = \frac{90}{7}$		$Z_j \rightarrow$	(1/7)	2	3	-1	X	X	
		Δ_j	$\underset{\uparrow}{\left(\frac{6}{7}\right)}$	0	0	0	X	X	
2	x_2	5/2	0	1	0	1/6	X	X	
3	x_3	5/2	0	0	1	1/2	X	X	
1	x_1	5/2	1	0	0	7/6	X	X	
$Z = \frac{30}{2} = 15$		$Z_j \rightarrow$	1	2	3	11/6	X	X	
		$\Delta_j \rightarrow$	0	0	0	-38/6	X	X	

$\Rightarrow x_1 = x_2 = x_3 = \frac{5}{2}$, max $Z = 15$ //

Q2 Use penalty method to max $Z = 3x_1 - x_2$ s.t.
$2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 3$, $x_2 \leq 4$, $x_1, x_2 \geq 0$

Sol^n std. form → Max $Z = 3x_1 - x_2 + 0s_1 + 0s_2 ~~- Ma_1~~ + 0s_3 - Ma_1$

s.t. $2x_1 + x_2 - s_1 + a_1 = 2$

$x_1 + 3x_2 + s_2 = 3$

$x_2 + s_3 = 4$

	$c_j \rightarrow$		3	-1	0	0	0	-M	
C_B	Basic Var.	x_B	x_1	x_2	s_1	s_2	s_3	a_1	Min Ratio
-M	a_1	2	$\boxed{2}$	1	-1	0	0	1	$1 \rightarrow$
0	s_2	3	1	3	0	1	0	0	3
0	s_3	4	0	1	0	0	1	0	
$Z = -2M$		$Z_j \rightarrow$	-2M	-M	M	0	0	-M	
		$\Delta_j \rightarrow$	$\underset{\uparrow}{(3+2M)}$	-1+M	-M	0	0	0	
3	x_1	1	1	1/2	-1/2	0	0	X	—
0	s_2	2	0	5/2	$\boxed{1/2}$	1	0	X	$4 \rightarrow$
0	s_3	4	0	1	0	0	1	X	—

z=3	~~x1~~ ~~s1~~ ~~s2~~	Zj→	3	3/2	-3/2	0	0	X
		Δj→	0	-5/2	$\boxed{3/2}$ ↑	0	0	X
3	x_1	3	1	3	0	1	0	X
0	s_1	4	0	5	1	2	0	X
0	s_2	4	0	1	0	0	1	X
Z = 9		Zj→	3	9	0	3	0	X
		Δj→	0	-10	0	-3	0	X

So, $x_1 = 3, x_2 = 0$, Max $Z = 9$

③ Max. $Z = ~~8x_2~~ 4x_1 + 5x_2 - 3x_3 + 50$

s.t. $x_1 + x_2 + x_3 = 10$, $x_1 - x_2 \geq 1$, $2x_1 + 3x_2 + x_3 \leq 40$

$x_1, x_2, x_3 \geq 0$.

Sol std form → Max $Z = 4x_1 + 5x_2 - 3x_3 + 0s_1 + 0s_2 - Ma_1 - Ma_2$

s.t. $x_1 + x_2 + x_3 ~~+ s_1~~ + a_1 = 10$

$x_1 - x_2 - s_1 + a_2 = 1$

$2x_1 + 3x_2 + x_3 + s_2 = 40$

cj → 4 5 -3 0 0 -M -M

C_B	Basic Var.	X_B	x_1	x_2	x_3	s_1	s_2	a_1	a_2	Min ratio
-M	a_1	10	1	1	1	0	0	1	0	10
-M	a_2	1	$\boxed{1}$	-1	0	-1	0	0	1	1 →
0	s_2	40	2	3	1	0	1	0	0	20
Z = -11M		Zj →	-2M	0	-M	M	0	-M	-M	
		Δj →	4+2M ↑	5	M-3	-M	0	0	0	
-M	a_1	9	0	$\boxed{2}$	1	1	0	1	X	9/2 →
4	x_1	1	1	-1	0	-1	0	0	X	—
0	s_2	38	0	5	1	2	1	0	X	38/5
Z = -9M+4		Zj →	4	-2M-4	-M	-M+4	0	-M	X	
		Δj →	0	2M+9 ↑	3+M	M-4	0	0	X	

5.	x_2	9/2	0	1	1/2	1/2	0	X	X
4	x_1	11/2	1	0	1/2	-1/2	0	X	X
0	s_2	31/2	0	0	-3/2	-1/2	1	X	X
Z = 89/2		Zj →	4	5	9/2	1/2	0	X	X
		Δj →	0	0	-15/2	-1/2	0	X	X

So, $x_1 = \frac{11}{2}$, $x_2 = \frac{9}{2}$, $z = \frac{89}{2} + 50$

$= \frac{189}{2}$

[illegible] artificial var isehi hai to prefer kiya h

④ Max $Z = ~~2x_1 + x_2~~ 5x_1 - 2x_2 + 3x_3$ s.t. $2x_1 + 2x_2 - x_3 \geq 2$, $3x_1 - 4x_2 \leq 3$,

$x_2 + 3x_3 \leq 5$, and $x_1, x_2, x_3 \geq 0$.

Sol std form → Max $Z = 5x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3 - Ma_1$

s.t. $2x_1 + 2x_2 - x_3 - s_1 + a_1 = 2$, $3x_1 - 4x_2 + s_2 = 3$,

$x_2 + 3x_3 + s_3 = 5$

cj → 5 -2 3 0 0 0 -M

C_B	B.V.	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Min ratio
-M	a_1	2	$\boxed{2}$	2	-1	-1	0	0	1	① →
0	s_2	3	3	-4	0	0	1	0	0	1
0	s_3	5	0	1	3	0	0	1	0	—
Z = -2M		Zj →	-2M	-2M	M	M	0	0	-M	
		Δj →	2M+5 ↑	2M-2	3-M	-M	0	0	0	
5	x_1	1	1	1	-1/2	-1/2	0	0	X	—
0	s_2	0	0	-7	$\boxed{3/2}$	3/2	1	0	X	0 →
0	s_3	5	0	1	3	0	0	1	X	5/3
Z = 5		Zj →	5	5	-5/2	-5/2	0	0	X	
		Δj →	0	-7	11/2 ↑	5/2	0	0	X	
5	x_1	1	1	-4/3	0	0	1/3	0	X	—
3	x_3	0	0	-14/3	1	1	2/3	0	X	—
0	s_3	5	0	$\boxed{15}$	0	3	0	1	X	5/15 →

$-20 - 42 = -\frac{62}{3}$

Z=5		Zj→	5	-62/3	3	3	11/3	0	X	
		Δj→	0	98/3	0	-3	-11/3	0	X	

5	x_1	13/9	1	0	0	-4/15	7/45	4/45	X	—
3	x_3	14/9	0	0	1	$\boxed{1/15}$	2/45	14/45	X	70/3 →
-2	x_2	1/3	0	1	0	-1/15	-2/45	1/15	X	—
Z = $\frac{101}{9}$		Zj→	5	-2	3	-11/15	53/15	56/45	X	
		Δj→	0	0	0	11/15 ↑	-53/45	-56/45	X	

5	x_1	23/3	1	0	4	0	1/3	4/3	X
0	s_1	70/3	0	0	15	1	2/3	14/3	X
-2	x_2	5	0	1	3	0	0	1	X
Z = $\frac{85}{3}$		Zj→	5	-2	14	0	5/13	14/3	X
		Δj→	0	0	-11	0	-5/13	-14/3	X

$\therefore x_1 = \frac{23}{3}, x_2 = 5, x_3 = 0$, max $Z = \frac{85}{3}$.

⑤ Max $Z = 6x_1 + 4x_2$ s.t. $2x_1 + 3x_2 \le 30$, $3x_1 + 2x_2 \le 24$, $x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$.

Sol^n, Std. form: Max $Z = 6x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 - Ma_1$
s.t. $2x_1 + 3x_2 + s_1 = 30$; $3x_1 + 2x_2 + s_2 = 24$,
$x_1 + x_2 - s_3 + a_1 = 3$

		Cj→	6	4	0	0	0	-M	
C_B	B.V.	X_B	x_1	x_2	s_1	s_2	s_3	a_1	Min Ratio
0	s_1	30	2	3	1	0	0	0	15
0	s_2	24	3	2	0	1	0	0	8
-M	a_1	3	$\boxed{1}$	1	0	0	-1	1	3 →
Z=-3M		Zj→	-M	-M	0	0	M	-M	
		Δj→	6+M ↑	4+M	0	0	-M	0	

0	s_1	24	0	1	1	0	2	X	12
0	s_2	15	0	-1	0	1	$\boxed{3}$	X	5 →
6	x_1	3	1	1	0	0	-1	X	—
Z=18		Zj	6	6	0	0	-6	X	
		Δj→	0	-2	0	0	6 ↑	X	

0	s_1	14	0	$\boxed{5/3}$	1	-2/3	0	X	14÷5/3 →
0	s_3	5	0	-1/3	0	1/3	1	X	—
6	x_1	8	1	2/3	0	1/3	0	X	0÷2/3
Z=48		Zj	6	4	0	2	0	X	
		Δj	0	0 ↑	0	-2	0	X	

$\boxed{x_1 = 8, x_2 = 0}$, $Z = 48$ //.

10/Aug/23

ALTERNATE OPTIMUM Sol^n

Max $Z = 6x_1 + 4x_2$ s.t. $2x_1 + 3x_2 \le 30$, $3x_1 + 2x_2 \le 24$, $x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$. Is the sol^n unique. If not give two diff. sol^ns.

1st sol^n

2nd sol^n.

Here element in Δj corresponding to x_2 is 0

⇒ Alternate sol^n exists.

(isme check kina h ki non-basic ~~elements~~ variables ke Δj ke corresponding ~~for~~ agr koi 0 h. to alternate sol^n exist)

C_B	B.V.	X_B	x_1	x_2	s_1	s_2	s_3	a_1	Min Ratio
4	x_2	42/5	0	1	3/5	-2/5	0	X	
0	s_3	39/5	0	0	1/5	1/5	1	X	
6	x_1	12/5	1	0	-2/5	3/5	0	X	
Z=48		Zj→	6	4	0	-2	0	X	
		Δj→	0	0	0	2	0	X	

→ Max $Z = 48$, $\boxed{x_1 = \frac{12}{5}, x_2 = \frac{42}{5}}$

$-2+\frac{9}{4} = \frac{11}{4}M$ $-\frac{7}{4}-\frac{7}{44}$ $-\frac{11-1}{44}$ $-\frac{22}{44}$ $-3+\frac{9}{2}$ $\frac{1}{2}-\frac{6}{11\times4}$

Q Min $z = 2x_1+9x_2+x_3$ s.t. $x_1+4x_2+2x_3 \geq 5$,
$3x_1+x_2+2x_3 \geq 4$, $x_1,x_2,x_3 \geq 0$.

Sol. std. form Max $z' = -2x_1-9x_2-x_3+0s_1+0s_2-Ma_1-Ma_2$

s.t. $x_1+4x_2+2x_3-s_1+a_1 = 5$

$3x_1+x_2+2x_3-s_2+a_2 = 4$

$c_j \to$ -2 -9 -1 0 0 -M -M

C_B	B.V.	x_B	x_1	x_2	x_3	s_1	s_2	a_1	a_2	Min Ratio
-M	a_1	5	1	$\boxed{4}$	2	-1	0	1	0	5/4 →
-M	a_2	4	3	1	2	0	-1	0	1	4
Z = -9M		$Z_j \to$	-4M	-5M	-4M	M	M	-M	-M	
		$\Delta_j \to$	4M∓2	5M∓9 ↑	4M-1	-M	-M	0	0	
-9	x_2	5/4	1/4	1	1/2	-1/4	0	X	0	5
-M	a_2	11/4	$\boxed{11/4}$	0	3/2	1/4	-1	X	1	1 →
$Z = -\frac{45}{4}-\frac{11}{4}M$		$Z_j \to$	$-\frac{9}{4}-\frac{11}{4}M$	-9	$-\frac{9}{2}-\frac{3}{2}M$	$\frac{9}{4}-\frac{1}{4}M$	M	X	-M	
		$\Delta_j \to$	$\frac{1}{4}+\frac{11}{4}M$ ↑	0	$\frac{3}{2}+\frac{3}{2}M$	$\frac{1}{4}M-\frac{9}{4}$	-M	X	0	
-9	x_2	1	0	1	-4/11	-3/11	1/11	X	X	11/4
-2	x_1	1	1	0	$\boxed{6/11}$	1/11	-4/11	X	X	11/6 →
Z = -11		$Z_j \to$	-2	-9	$-\frac{48}{11}$	-25/11	-1/11	X	X	
		Δ_j	0	0	37/11 ↑	-25/11	1/11	X	X	
-9	x_2	4/3	-2/3	1	0	-1/3	$\boxed{1/3}$	X	X	1 →
-1	x_3	11/6	11/6	0	1	~~-2/3~~ 1/6	-2/3	X	X	—
Z =		$Z_j \to$	25/6	-9	-1	17/6	-7/3	X	X	
		$\Delta_j \to$	-37/6	0	0	-17/6	7/3 ↑	X	X	
0	s_2	1	-2	3	0	-1	1	X	X	
-1	x_3	5/2	1/2	2	1	-1/2	0	α	α	
Z = -5/2		Z_j	-1/2	-2	-1	1/2	0	α	α	
		$\Delta_j \to$	-3/2	-7	0	-1/2	0	α	α	

$-1+\frac{9}{2}$

$x_1 = 0$, $x_2 = 0$, $x_3 = 5/2$

Max $z = -\frac{5}{2}$

Min $z = 5/2$

11/Aug

TWO PHASE SIMPLEX METHOD :-

→ convert constraints into std form as Big-M-method

Q Use 2-phase simplex method to solve the problem min $Z = x_1-2x_2-3x_3$
s.t. $-2x_1+x_2+3x_3 = 2$, $2x_1+3x_2+4x_3 = 1$,
$x_1, x_2, x_3 \geq 0$

Sol Std. form: max $z' = -x_1+2x_2+3x_3$

s.t. $-2x_1+x_2+3x_3 ~~-s_1~~ +a_1 = 2$

$2x_1+3x_2+4x_3 ~~-s_2~~ +a_2 = 1$

PHASE-I → Auxilary LPP is

Max $z^* = 0x_1+0x_2+0x_3-a_1-a_2$

in obj. fon:
- Decision variable → $x_1, x_2, x_3, \ldots$ } all coeff = 0
- Slack / surplus → $s_1, s_2, \ldots$ } all coeff = 0
- artificial variable coeff → -1

Constraints same

$-2x_1+x_2+3x_3+a_1 = 2$

$2x_1+3x_2+4x_3+a_2 = 1$

$x_1, x_2, x_3 \geq 0$

Phase I

$c_j \to$ 0 0 0 -1 -1

C_B	B.V.	x_B	x_1	x_2	x_3	a_1	a_2	Min Ratio
-1	a_1	2	-2	1	3	1	0	2/3
-1	a_2	1	2	3	$\boxed{4}$	0	1	1/4 →
Z = -3		$Z_j \to$	0	-4	-7	-1	-1	
		$\Delta_j \to$	0	4	7 ↑	0	0	

$R_2 \to R_2/4$ $R_1 \to R_1 - 3R_2$

-1	a_1	5/4	-7/2	-5/4	0	1	-3/4
0	x_3	1/4	1/2	3/4	1	0	1/4
$Z^* = -\frac{5}{4}$		$Z_j \rightarrow$	7/2	5/4	0	-1	3/4
		$\Delta_j \rightarrow$	-7/2	-5/4	0	0	-7/4

Since all $\Delta_j \leq 0$ & optimum basic feasible solⁿ to the auxilary LPP has been attained. But at the same time max Z^* is -ve & the artificial variable a_1 appears in the optimum basis at a +ve level.

Hence, the ~~opt~~ original problem doesn't have any ~~physical~~ feasible solⁿ.

Here, there's no need to enter into phase 2

→ Condⁿs observed after Phase 1 -

① When Max $Z^* < 0$ & atleast 1 artificial variable appear in the optimum basis (B.V.) at ~~at~~ a +ve level.

In this case given problem doesn't have any solⁿ.

② When Max $Z^* = 0$ & atleast 1 artificial variable appear in the optimum basis (B.V) at 0 level.

In this case proceed to phase 2.

③ Max $Z^* = 0$ & no artificial variable appear in the optimum Basis.

In this case also proceed to phase 2.

Q Min $Z = \frac{15}{2}x_1 - 3x_2$ s.t. $3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 + x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$

Sol Std form → Max $Z' = -\frac{15}{2}x_1 + 3x_2$

s.t. $3x_1 - x_2 - x_3 - s_1 + a_1 = 3$,
$x_1 - x_2 + x_3 - s_2 + a_2 = 2$.

Phase1 → Auxilary LPP is

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 - a_1 - a_2$$

s.t. $3x_1 - x_2 - x_3 - s_1 + a_1 = 3$,
$x_1 - x_2 + x_3 - s_2 + a_2 = 2$

		$c_j \rightarrow$	0	0	0	0	0	-1	-1	
C_B	B.V.	X_B	x_1	x_2	x_3	s_1	s_2	a_1	a_2	Min Ratio
-1	a_1	3	$\boxed{3}$	-1	-1	-1	0	1	0	1 →
-1	a_2	2	1	-1	1	0	-1	0	1	2
$Z^* = -5$		$Z_j \rightarrow$	-4	2	0	1	1	-1	-1	
		$\Delta_j \rightarrow$	4	-2	0	-1	-1	0	0	
		$R_1 \rightarrow R_1 \div 3$	↑	$R_2 \rightarrow R_2 - R_1$						
0	X_1	1	1	-1/3	-1/3	-1/3	0	1/3	0	—
-1	a_2	1	0	-2/3	$\boxed{4/3}$	1/3	-1	-1/3	1	3/4 →
$Z^* = -1$		$Z_j \rightarrow$	0	2/3	-4/3	-1/3	1	1/3	-1	
		$\Delta_j \rightarrow$	0	-2/3	4/3	1/3	-1	-4/3	0	
		$R_2 \rightarrow R_2 \times 3/4$	$R_1 \rightarrow R_1 + \frac{1}{3}R_2$		↑					
0	X_1	5/4	1	-1/2	0	-1/4	-1/4	1/4	1/4	
0	X_3	3/4	0	-1/2	1	1/4	-3/4	-1/4	3/4	
$Z^* = 0$		$Z_j \rightarrow$	0	0	0	0	0	0	0	
		$\Delta_j \rightarrow$	0	0	0	0	0	-1	-1	

all $\Delta_j \leq 0$ & no artificial variable, appear in optimum basis.

So, we'll enter in phase 2.

max z's value

max z's value

Phase2

		Cj →	-15/2	3	0	0	0
CB	B.V.	XB	X1	X2	X3	s1	s2
-15/2	X1	5/4	1	-1/2	0	-1/4	-1/4
0	X3	3/4	0	-1/2	1	1/4	-3/4
						15/8	15/8
Z' = -75/8		Zj →	-15/2	15/4	0	-15/8	-15/8
		Δj →	0	-3/4	0		

Here all $\Delta_j \le 0$. so, the optimum soln has been attained.

$$\text{Max } z' = -\frac{75}{8} \Rightarrow \text{Min } z = \frac{75}{8}$$

$$x_1 = \frac{5}{4};\ x_2 = 0;\ x_3 = \frac{3}{4}$$

Note: for case 2
Phase 1 me jo A.V. bchega use phase 2 me bhi use krege or doosre ko hta denge.

Assignment

2-phase Method Questions

① Max $z = 3x_1 - x_2$ s.t. $2x_1 + x_2 \ge 2$, $x_1 + 3x_2 \le 2$, $x_2 \ge 4$, and $x_1, x_2 \ge 0$.

Soln Phase1 → Auxiliary eqn is

$$\text{Max } z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 - a_1 - a_2$$

s.t. $2x_1 + x_2 - s_1 + a_1 = 2$

$x_1 + 3x_2 + s_2 = 2$

$x_2 - s_3 + a_2 = 4$

$x_1, x_2, s_1, s_2, s_3 \ge 0$.

		Cj →	0	0	0	0	0	-1	-1	
CB	B.V.	XB	x1	x2	s1	s2	s3	a1	a2	Min Ratio
-1	a1	2	$\boxed{2}$	1	-1	0	0	1	0	1 →
0	s2	2	1	3	0	1	0	0	0	2
-1	a2	4	0	1	0	0	-1	0	1	–
Z = -6		Zj →	-2	-2	1	0	1	-1	-1	
		Δj →	2	2	-1	0	-1	0	0	
			↑							
0	X1	1	1	1/2	-1/2	0	0	1/2	0	2
0	s2	1	0	$\boxed{5/2}$	1/2	1	0	-1/2	0	2/5 →
-1	a2	4	0	1	0	0	-1	0	1	4
z* = -4		Zj →	0	-1	0	0	1	0	-1	
		Δj →	0	1	0	0	-1	-1	0	
				↑						
0	X1	4/5	1	0	-3/5	-1/5	0	3/5	0	
0	X2	2/5	0	1	1/5	2/5	0	-1/5	0	
-1	a2	18/5	0	0	-1/5	-2/5	-1	1/5	1	
z* = -18/5		Zj →	0	0	1/5	2/5	1	-1/5	-1	
		Δj →	0	0	-1/5	-2/5	-1		0	

Phase-2. No soln exists.

Q2 Max $z = 5x_1 + 8x_2$ s.t. $3x_1 + 2x_2 \ge 3$, $x_1 + 4x_2 \ge$
$x_1 + x_2 \le 5$, $x_1, x_2 \ge 0$.

Soln Phase1 : Auxiliary eqn is

$$\text{Max } z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_2 - a_1 - a_2$$

s.t. $3x_1 + 2x_2 - s_1 + a_1 = 3$

$x_1 + 4x_2 - s_2 + a_2 = 4$

$x_1 + x_2 + s_3 = 5$

$x_1, x_2, s_1, s_2, s_3 \ge 0$.

C_B	B.V.	X_B	x_1	x_2	s_1	s_2	s_3	a_1	a_2	Min Ratio
	$c_j \to$		0	0	0	0	0	-1	-1	
-1	a_1	3	3	2	-1	0	0	1	0	3/2
-1	a_2	4	1	$\boxed{4}$	0	-1	0	0	1	1 →
0	s_3	5	1	1	0	0	1	0	0	5
$z^* = -1$		$Z_j \to$	-4	-6	1	1	0	-1	-1	
		$\Delta_j \to$	4	6 ↑	-1	-1	0	0	0	
-1	a_1	1	$\boxed{5/2}$	0	-1	1/2	0	1	x	2/5 →
0	x_2	1	1/4	1	0	-1/4	0	0	1/4	4
0	s_3	4	3/4	0	0	1/4	1	0	-1/4	$4 \div \frac{3}{4}$
$z^* = -1$		$Z_j \to$	-5/2	0	1	-1/2	0	-1		
		$\Delta_j \to$	5/2 ↑	0	-1	1/2	0	0		
0	x_1	2/5	1	0	-2/5	1/5	0	x	x	
0	x_2	9/10	0	1	1/10	-3/10	0	x	x	
0	s_3	37/10	0	0	3/10	1/10	1	x	x	
$z^* = 0$		$Z_j \to$	0	0	0	0	0			
		$\Delta_j \to$	0	0	0	0	0			

Phase-2 → $\text{Max } z = 5x_1 + 8x_2 + 0s_1 + 0s_2 + 0s_3$

C_B	B.V.	X_B	x_1	x_2	s_1	s_2	s_3	Min Ratio
	$c_j \to$		5	8	0	0	0	
5	x_1	2/5	1	0	-2/5	$\boxed{1/5}$	0	2 →
8	x_2	9/10	0	1	1/10	-3/10	0	—
0	s_3	37/10	0	0	3/10	1/10	1	37
$z' = 46/5$		$Z_j \to$	5	8	-6/5	-7/5	0	
		$\Delta_j \to$	0	0	6/5	7/5 ↑	0	
0	s_2	2	5	0	-2	1	0	—
8	x_2	15/10	15/20	1	-1/2	0	0	—
0	s_3	23/10	-1/2	0	$\boxed{1/2}$	0	1	7 →
$z' = 12$		$Z_j \to$	12	8	-4	0	0	
		$\Delta_j \to$	-7	0	4 ↑	0	0	
0	s_2	16	3	0	0	1	4	
8	x_2	5	1	1	0	0	1	
0	s_1	7	-1	0	1	0	2	
$z' = 40$		$Z_j \to$	8	8	0	0	8	
		$\Delta_j \to$	-8	-8	0	0	-8	

so, Max $z = 40$, $x_1 = 0, x_2 = 5$.

Q3 $\text{Max } z = x_1 + 15x_2 + 2x_3 + 5x_4$

s.t. $3x_1 + 2x_2 + 4x_3 + x_4 \le 6$,

$2x_1 + x_2 + x_3 + 5x_4 \le 4$,

$2x_1 + 6x_2 - 8x_3 + 4x_4 = 0$,

$x_1 + 3x_2 - 4x_3 + 3x_4 = 0$, $\quad x_1, x_2, x_3, x_4 \ge 0$.

Sol: Phase I → Auxiliary eqs

$\text{Max } z^* = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0s_1 + 0s_2 - a_1 - a_2$

s.t. $3x_1 + 2x_2 + 4x_3 + x_4 + s_1 = 6$

$2x_1 + x_2 + x_3 + 5x_4 + s_2 = 4$

$2x_1 + 6x_2 - 8x_3 + 4x_4 + a_1 = 0$

$x_1 + 3x_2 - 4x_3 + 3x_4 + a_2 = 0$

C_B	B.V.	X_B	x_1	x_2	x_3	x_4	s_1	s_2	a_1	a_2	Min Ratio
	$c_j \to$		0	0	0	0	0	0	-1	-1	
0	s_1	6	3	$\boxed{2}$	4	1	1	0	0	0	3
0	s_2	4	2	1	1	5	0	1	0	0	4
-1	a_1	0	2	$\boxed{6}$	-8	4	0	0	1	0	0 →
-1	a_2	0	1	3	-4	3	0	0	0	1	0
$z^* = 0$		$Z_j \to$	-3	-9	12	-7	0	0	-1	-1	
		$\Delta_j \to$	3	9 ↑	-12	7	0	0	0	0	
~~0~~	~~x_2~~	~~3~~	~~3/2~~	~~1~~	~~2~~	~~1/2~~	~~1/2~~	~~0~~	~~0~~	~~0~~	~~6~~
~~0~~	~~s_2~~	~~1~~	~~1/2~~	~~0~~	~~-1~~	~~$\boxed{9/2}$~~	~~-1/2~~	~~1~~	~~0~~	~~0~~	~~2/9~~ →
~~-1~~	~~a_1~~	~~-18~~	~~-7~~	~~0~~	~~-20~~	~~1~~	~~3~~	~~0~~	~~1~~	~~0~~	~~—~~
~~-1~~	~~a_2~~	~~-9~~	~~-7/2~~	~~0~~	~~-10~~	~~3/2~~	~~-3/2~~	~~0~~	~~0~~	~~1~~	~~—~~

5 2/3 (15-2)/2 → 15/3 -8.4/63 -15/2 3·2/3 -4·9 ·4·2-9 -3+2 -4/3
4+8/3 →12+8 -12/2 9 3-4/2 2/6 → 2 - 2 0+1 2/7 1+4/3 3.

~~$z^* = 27$~~		~~z_j~~	~~21/2~~	~~0~~	~~30~~	~~-5/2~~	~~-3/2~~	~~0~~	~~-1~~	~~-1~~
		~~Δ_j~~	~~-21/2~~	~~0~~	~~-30~~	~~5/2~~ ↑	~~3/2~~	~~0~~	~~0~~	~~0~~
~~0~~	~~x_2~~									
~~0~~	~~x_4~~	~~2/9~~	~~9~~	~~0~~	~~-2/9~~	~~1~~	~~-9~~	~~2/9~~	~~0~~	~~0~~
~~-1~~	~~a_1~~	~~-45/2~~	~~-16~~	~~0~~						
~~-1~~	~~a_2~~									

0	s_1	6	7/3	0	20/3	-1/3	1	0	α	0	
0	s_2	4	5/3	0	7/3	13/3	0	1	α	0	4÷13/3
0	x_2	0	1/3	1	-4/3	2/3	0	0	α	0	0
-1	a_2	0	0	0	0	$\boxed{1}$	0	0	α	1	0 →
$z^* = 0$		z_j	0	0	0	-1	0	0	α	-1	
		Δ_j	0	0	0	1 ↑	0	0	α	0	

0	s_1	6	7/3	0	20/3	0	1	0	α	α
0	s_2	4	5/3	0	7/3	0	0	1	α	α
0	x_2	0	1/3	1	-4/3	0	0	0	α	α
0	x_4	0	0	0	0	1	0	0	α	α
$z^* = 0$		z_j	0	0	0	0	0	0	α	α
		Δ_j	0	0	0	0	0	0	α	α

Phase2 →

$\text{Max } z' = x_1 + 15x_2 + 2x_3 + 5x_4 + 0s_1 + 0s_2$

		c_j	1	15	2	5	0	0	
C_B	B.V.	x_B	x_1	x_2	x_3	x_4	s_1	s_2	Min Ratio
0	s_1	6	7/3	0	$\boxed{20/3}$	0	1	0	$6 \times \frac{3}{20}$ →
0	s_2	4	5/3	0	7/3	0	0	1	$4 \times \frac{3}{7}$
15	x_2	0	1/3	1	-4/3	0	0	0	—
5	x_4	0	0	0	0	1	0	0	←
$z' = 0$		z_j	5	15	-20	5	0	0	
		Δ_j	4	0	22 ↑	0	0	0	

5: Select the smallest Quantity amongst the cells marked with

of the closed

of to the

(NOTE → Agr phase2 me B.V. me koi a.v. aye to uske corresponding C_B is always 0.)

2	x_3	0.9	0.35	0	1	0	0.15	0
0	s_2	1.9	0.85	0	0	0	-0.35	1
15	x_2	1.2	0.8	1	0	0	0.2	0
5	x_4	0	0	0	0	1	0	0
$z' = 19.8$		z_j	~~2.89~~ 12.7	15	2	5	3.3	0
		Δ_j	~~-2.89~~ 12.7	0	0	0	-3.3	0

Max $z' = 19.8$, $x_1 = 0$, $x_2 = 1.2$, $x_3 = 0.9$,
$x_4 = 0$.

Q4 Minimize $z = x_1 - 2x_2 - 3x_3$ s.t. $-2x_1 + x_2 + 3x_3 = 2$,
$2x_1 + 3x_2 + 4x_3 = 1$, $x_1, x_2, x_3 \geq 0$.

Sol Std form Max $Z = -x_1 + 2x_2 + 3x_3 - a_1 - a_2$

Phase1 Auxiliary eq → Max $z^* = 0x_1 + 0x_2 + 0x_3 - a_1 - a_2$

s.t. $-2x_1 + x_2 + 3x_3 + a_1 = 2$

$2x_1 + 3x_2 + 4x_3 + a_2 = 1$

		c_j	0	0	0	-1	-1	
C_B	B.V.	x_B	x_1	x_2	x_3	a_1	a_2	Min Ratio
-1	a_1	2	-2	1	1	1	0	
-1	a_2	1	2	3	4	0	1	
$z^* = -3$								

Done Before in C/w

Q5 Max $z = 3x_1 + 2x_2 + x_3 + 4x_4$ s.t.
$4x_1 + 5x_2 + x_3 - 3x_4 = 5$, $2x_1 - 3x_2 - 4x_3 + 5x_4 = 7$,
$x_1 + 4x_2 + 2.5x_3 - 4x_4 = 6$, $x_1, x_2, x_3 \geq 0$.

Sol ~~A~~ Phase1 →

Auxiliary eqⁿ is

Max $Z^* = 0x_1 + 0x_2 + 0x_3 + 0x_4 - a_1 - a_2 - a_3$

s.t. $4x_1 + 5x_2 + x_3 - 3x_4 + a_1 = 5$,

$2x_1 - 3x_2 - 4x_3 + 5x_4 + a_2 = 7$,

$x_1 + 4x_2 + 2.5x_3 - 4x_4 + a_3 = 6$.

$\frac{4}{5}\times\frac{1}{34}$ 17 $\frac{9}{85}$ $-\frac{21}{5}\times\frac{9}{34}$ $\frac{2170+22}{45}$ $4-\frac{98}{45}$ $1+\frac{54}{85}$ 2.2 25+9

		c_j→	0	0	0	0	-1	-1	-1	
C_B	B.V.	x_B	x_1	x_2	x_3	x_4	a_1	a_2	a_3	Min ratio
-1	a_1	5	$\boxed{4}$	$\boxed{5}$	1	-3	1	0	0	5/5 →
-1	a_2	7	2	-3	-4	5	0	1	0	7/3
-1	a_3	6	1	4	2.5	-4	0	0	1	6/4
$Z^* = -18$		z_j→	-7	-6	-.5	2	-1	-1	-1	
		Δ_j→	7	6	.5	-2	0	0	0	
			↑	↑						
0	x_2	1	4/5	1	1/5	-3/5	X	0	0	—
-1	a_2	4	-2/5	0	-21/5	34/9	X	1	0	→
-1	a_3	2	-11/5	0	1.7	-8/5	X	0	1	—
$Z^* = -6$		z_j→	13/5	0	2.5	-48/45	X	-1	-1	·29-3.1
		Δ_j→	-13/5	0	-2.5	98/45	X	0	0	$\frac{4}{5}+\frac{27}{425}$ ·86
						↑				
0	x_2	139/85	·86	1	·87	0	X	X	0	·2
0	x_4	18/17	9/85	0	-189/120	1	X	X	0	
-1	a_3	3.69				0	X	X	1	
$Z^* = -3.69$		z_j→				0	X	X	-1	$2+\frac{90}{170}$
						0	X	X	0	

$\frac{149}{31}$

0	x_1	1.25	1	1.25	0.25	-0.75	0	0	—
-1	a_2	4.5	0	-5.5	-4.5	6.5	1	0	·69 →
-1	a_3	4.75	0	2.75	2.25	-3.25	0	1	—
$Z^* = -9.5$		z_j→	0	2.75	2.25	-3.25	-1	-1	
		Δ_j→	0	-2.75	-2.25	3.25	0	0	
						↑			
0	x_1	1.76	1	·62	-·27	~~-0.26~~	0α20		
0	x_4	·69	0	-·85	-·70	~~-0.69~~	1α<0		
-1	a_3	7	0	0	0	~~0~~	0α<1		
$Z^* = -7$		z_j→	0	0	0	0	0α<1		
		Δ_j→	0	0	0	0	0α<0		

No Soln //

$\frac{3.25}{2}$ $1\times\frac{1}{3}$

⑥ Max $Z = 5x_1 - 2x_2 + 3x_3$ s.t. $2x_1 + 2x_2 - x_3 \geq 2$
$3x_1 - 4x_2 \leq 3$, $x_2 + 3x_3 \leq 5$, $x_1, x_2, x_3, x_4 \geq 0$

Sol Phase1 → A.E, Max $Z^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 + 0s_3 - a_1$
S.t. → $2x_1 + 2x_2 - x_3 - s_1 + a_1 = 2$, $3x_1 - 4x_2 + s_2 = 3$
$x_2 + 3x_3 + s_3 = 5$.

		c_j→	0	0	0	0	0	0	-1	
C_B	B.V.	x_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Min
-1	a_1	2	$\boxed{2}$	2	-1	-1	0	0	1	1 →
0	s_2	3	3	4	0	0	1	0	0	1
0	s_3	5	0	2	3	0	0	1	0	—
$Z^* = -2$		z_j→	-2	-2	1	1	0	0	-1	
		Δ_j→	2	2	-1	-1	0	0	1	
			↑							
0	x_1	1	1	1	-1/2	-1/2	0	0	~~1/2~~	
0	s_2	0	0	-7	3/2	3/2	1	0	α	
0	s_3	5	0	1	3	0	0	1	α	
$Z^* = 0$		z_j→	0	0	0	0	0	0	α	
		Δ_j→	0	0	0	0	0	0	α	

Phase2 → A.E → Max $Z' = 5x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3$

		c_j	5	-2	3	0	0	0	
C_B	B.V.	x_B	x_1	x_2	x_3	s_1	s_2	s_3	Min
5	x_1	1	1	1	-1/2	-1/2	0	0	—
0	s_2	0	0	-7	$\boxed{3/2}$	3/2	1	0	0 →
0	s_3	5	0	1	3	0	0	1	5/3
$Z' = 5$		z_j→	5	5	-5/2	-5/2	0	0	
		Δ_j→	0	-7	11/2	5/2	0	0	
					↑				
5	x_1	1	1	-4/3	0	0	1/3	0	→
3	x_3	0	0	-14/3	1	1	2/3	0	←
0	s_3	5	0	$\boxed{15}$	0	-3	-2	1	5/15 →

$\frac{1}{20}$

Z' = 5	zj→		5	62/3	3	3	11/3	0
	Δj→		0	2/3	0	-3	-11/3	0
				↑				
5	x1	13/9	1	0	0	-4/45	7/45	4/45	—
3	x3	14/9	0	0	1	$\boxed{1/15}$	2/45	14/45	→
-2	x2	1/3	0	1	0	-1/5	-2/15	1/15	—
Z' =		345/9	5	-2	3	-17/45	63/45	56/15
			0	0	0	17/45	-63/45	-56/45
						↑		
5	x1	69/9	1	0	4	0	1/3	4/3
140	s1	210/9	0	0	0	1	2/3	14/3
-2	x2	5	0	1	1/5	0	0	1
Z' = 85/3	zj→		5	-2	14	0	5/3	14/3
	Δj→		0	0	-11	0	-5/3	-14/3

So, $Z' = \frac{85}{3}$, $x_1 = \frac{69}{9}$, $x_2 = 5$, $x_3 = 0$

87 Max $z = 2x_1 + 3x_2 + 5x_3$ s.t. $3x_1 + 10x_2 + 5x_3 \le 15$, $x_1 + 2x_2 + x_3 \ge 4$, $33x_1 - 10x_2 + 9x_3 \le 33$, $x_1, x_2, x_3 \ge 0$

Soln Phase I → A.E → Max $z^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 + 0s_3 - a_1$

s.t. $3x_1 + 10x_2 + 5x_3 + s_1 = 15$

$x_1 + 2x_2 + x_3 - s_2 + a_1 = 4$

$33x_1 - 10x_2 + 9x_3 + s_3 = 33$

		cj→	0	0	0	0	0	0	-1	
CB	B.V.	XB	x1	x2	x3	s1	s2	s3	a1	Min
0	s1	15	3	$\boxed{10}$	5	1	0	0	0	15/10 →
-1	a1	4	1	2	1	0	-1	0	1	4/2
0	s3	33	33	-10	9	0	0	1	0	—
z* = -4	zj→		-1	-2	-1	0	1	0	-1	
	Δj→		1	2	1	0	-1	0	0	
				↑						
0	x2	3/2	3/10	1	1/2	1/10	0	0	0	5
-1	a1	1	2/5	0	0	-1/5	-1	0	1	5/2
0	s3	48	$\boxed{36}$	0	14	1	0	1	0	48/36 →
z* = -1	zj→		-2/5	0	0	1/5	1	0	-1	
	Δj→		2/5	0	0	-1/5	-1	0	0	
			↑							
0	x2	4/10	0	1	.38	-.09	0	-.008	0	
-1	a1	7/15	0	0	-7/45	-.21	-1	-1/36	1	
0	x1	4/3	1	0	7/18	.02	0	1/36	0	
Z = -7/15	zj		0	0	7/45	.21	1	1/36	-1	
	Δj		0	0	-7/45	-.21	-1	-1/36	0	

NO soln exist.

88 Max $500x_1 + 1400x_2 + 900x_3$ s.t.

$x_1 + x_2 + x_3 = 100$, $12x_1 + 35x_2 + 15x_3 \ge 25$,

$8x_1 + 3x_2 + 4x_3 \ge 6$, $x_1, x_2, x_3 \ge 0$.

Soln Phase I → A.E → Max $Z^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 - a_1 - a_2 - a_3$

s.t. → $x_1 + x_2 + x_3 + a_1 = 100$; $12x_1 + 35x_2 + 15x_3 - s_1 + a_2 = 25$

$8x_1 + 3x_2 + 4x_3 - s_2 + a_3 = 6$

		cj→	0	0	0	0	0	-1	-1	-1	
CB	B.V.	XB	x1	x2	x3	s1	s2	a1	a2	a3	Min
-1	a1	100	1	1	1	0	0	1	0	0	100
-1	a2	25	12	$\boxed{35}$	15	-1	0	0	1	0	25/35 →
-1	a3	6	8	3	4	0	-1	0	0	1	3
Z* = -131	zj→		-21	-39	-20	1	1	-1	-1	-1	
	Δj→		21	39	20	-1	-1	0	0	0	
				↑							
-1	a1	695/7	23/35	0	4/7	1/35	0	1	α	0	151.08
0	x2	5/7	12/35	1	3/7	-1/35	0	0	α	0	2.08
-1	a3	3.85	6.97	0	23/14	.085	-1	0	α	1	0.55 →
Z* → -103.14	zj→		-7.63	0	-3.31	-.114	1	-1	α	-1	
	Δj→		7.63	0	3.31	.114	-1	0	α	0	
			↑								

-1	a_1	98.42	0	0	·315	·0205	0.09	1	✓
0	x_2	0.52	0	1	-.295	-0.0328	0.05	0	✓
0	x_1	0.55	1	0	$\boxed{·389}$	-0.0123	-0.14	0	✓ →
Z = -98.42		z_j →	0	0	-·3156	-0.0205	-0.094	-1	✓
		Δ_j →	0	0	·3156 ↑	0.0205	0.094	0	✓

-1	a_1							
0	X_2							
0	X_3	1.413	2.57	0	1	-·0316	·359	0

Q. Max. $z = 5x_1 + 2x_2$ s.t. $6x_1 + x_2 \geq 6$, $4x_1 + 3x_2 \geq 12$, $x_1 + 2x_2 \geq 4$, $x_1, x_2 \geq 0$.

Sol Phase I → Max $Z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 - a_1 - a_2 - a_3$

s.t. $6x_1 + x_2 - s_1 + a_1 = 6$

$4x_1 + 3x_2 - s_2 + a_2 = 12$

$x_1 + 2x_2 - s_3 + a_3 = 4$

		c_j →	0	0	0	0	0	-1	-1	-1	
C_B	B.V.	X_B	x_1	x_2	S_1	S_2	S_3	a_1	a_2	a_3	Min Ratio
-1	a_1	6	$\boxed{6}$	1	-1	0	0	1	0	0	1 →
-1	a_2	12	4	3	0	-1	0	0	1	0	3
-1	a_3	4	1	2	0	0	-1	0	0	1	4
$Z^* = -22$		z_j →	-11	-6	1	1	1	-1	-1	-1	[illegible]
		Δ_j →	11 ↑	6	-1	-1	-1	0	0	0	[illegible]
0	x_1	1	1	1/6	-1/6	0	0	X	0	0	6
-1	a_2	8	0	7/3	2/3	-1	0	X	1	0	24/7
-1	a_3	3	0	$\boxed{11/6}$	1/6	0	-1	X	0	1	18/11 →
$Z^* = -11$		z_j →	0	-25/6	1/2	1	1	X	-1	-1	
		Δ_j →	0	25/6 ↑	-1/2	-1	-1	X	0	0	

[illegible]

[illegible]

			x_1	x_2	s_1	s_2	s_3	a_1	a_2	a_3	Min Ratio
0	x_1	8/11	1	0	-2/11	0	$\boxed{1/11}$	∝	0	∝	$\frac{8}{11} \div \frac{1}{11}$ = 8
-1	a_2	46/11	0	0	19/18	-1	$\boxed{14/11}$	∝	1	∝	138/14 = 9.857 →
0	x_2	18/11	0	1	1/11	0	-6/11	∝	0	∝	—
$Z^* = -\frac{138}{11}$		z_j →	0	0	19/18	1	-14/11	∝	-1	∝	
			0	0	-19/18	-1	14/11 ↑	∝	0	∝	
0	x_1	.428	1	0	-.214	.071	0	∝	∝	∝	
0	S_3	3.28	0	0	.357	-.785	1	∝	∝	∝	
0	x_2	3.42	0	1	.285	.4286	0	∝	∝	∝	
$Z^* = 0$		z_j →	0	0	0	0	0	∝	∝	∝	
		Δ_j →	0	0	0	0	0	∝	∝	∝	

Phase 2 → Max $Z' = 5x_1 + 2x_2 + 0S_1 + 0S_2 + 0S_3$

		c_j →	5	2	0	0	0	
C_B	B.V.	X_B	x_1	x_2	S_1	S_2	S_3	Min Ratio
5	x_1	0.428	1	0	-.214	.071	0	—
0	S_3	3.28	~~1~~0	0	.357	-.785	1	9.2 →
2	X_2	3.4286	~~6~~0	1	.285	.4286	0	12
$Z' = $ ~~52~~.9		z_j →	4.35	2	-0.5	-0.5	0	
		Δ_j →	~~38~~0	0	~~-136/11~~ 0.5 ↑	0.5	0	

~~Max Z = 52, $x_1 = 8$, $x_2 = 6$.~~

5	x_1	2.4	1	0	0	-.4	.6	—
0	S_1	9.2	0	0	1	-2.2	2.8	—
2	X_2	0.8	0	1	0	.2	-.8	4 →
$Z' = 13.6$		z_j →	5	2	0	-1.6	1.4	
		Δ_j →	0	0	0	1.6 ↑	-1.4	
5	x_1	4	1	2	0	0	-1	—
0	S_1	18	0	11	1	0	-6	—
0	S_2	4	0	5	0	1	-4	—
$Z' = 20$		z_j →	5	10	0	0	-5	
		Δ_j →	0	-8	0	0	5 ↑	

Variable S_3 should enter into the basis, but all coefficients in S_3 column are -ve or zero.
So, S_3 can't be entered into basis.
hence soln to given problem is unbounded.

Sol. Max $Z = 60{,}000x_1 + 12000x_2$ s.t. $9000x_1 + 12000x_2 \leq 720$
$x_1 \geq 2$, ~~3~~ $x_2 \geq 3$; $x_1, x_2 \geq 0$.

Phase I → A.E → Max $Z^* =$ ~~60,000~~ $0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 - a_1 - a_2$
s.t. $9000x_1 + 12000x_2 + s_1 = 720{,}000$
$x_1 - s_2 + a_1 = 2$
$x_2 - s_3 + a_2 = 3$

		cj →	0	0	0	0	0	-1	-1	
C_B	B.V.	x_B	x_1	x_2	s_1	s_2	s_3	a_1	a_2	Min Ratio
0	S_1	7,20,000	9000	12000	1	0	0	0	0	$\frac{720}{9}$
-1	a_1	2	$\boxed{1}$	0	0	-1	0	1	0	2 →
-1	a_2	3	0	1	0	0	-1	0	1	—
$Z^* = -5$		Zj →	-1	-1	0	1	1	-1	-1	
		Δj →	1	1	0	-1	-1	0	0	
			↑							
0	S_1	7,02000	0	12000	-1	9000	0	α	0	$\frac{702}{12}$
0	X_1	2	1	0	0	-1	0	α	0	—
-1	a_2	3	0	$\boxed{1}$	0	0	-1	α	1	3 →
$Z^* = -3$		Zj →	0	-1	0	0	1	α	-1	
		Δj →	0	1	0	0	-1	α	0	
				↑						
0	S_1	666,000	0	0	1	9000	12000	α	α	
0	X_1	2	1	0	0	-1	0	α	α	
0	X_2	3	0	1	0	0	-1	α	α	
$Z^* = 0$		Zj →	0	0	0	0	0	α	α	
		Δj →	0	0	0	0	0	α	α	

Phase 2 → Max $Z' = 60{,}000x_1 + 12{,}000x_2 + 0s_1 + 0s_2 + 0s_3$

		cj →	60,000	12,000	0	0	0	
C_B	B.V.	X_B	x_1	x_2	S_1	S_2	S_3	Min ratio
~~60,000~~ 0	S_1	6,66,000	0	0	1	$\boxed{9000}$	12000	666 →
60,000	X_1	2	1	0	0	-1	0	—
12,000	X_2	3	0	1	0	0	-1	—
$Z^* = 1{,}56{,}000$		Zj →	60,000	12,000	0	-60,000	-12000	
		Δj →	0	0	0	60,000	12000	
						↑		
0	S_2	74	0	0	$\frac{1}{9000}$	1	$\frac{4}{3}$	
60,000	X_1	76	1	0	$\frac{1}{9000}$	0	$\frac{4}{3}$	
12,000	X_2	3	0	1	0	0	-1	
$Z' = 4{,}596{,}000$		Zj →	60,000	12000	6.66	0	68,000	
		Δj →	0	0	-6.66	0	-68000	

Max $Z = 4596{,}000$
$X_1 = 76$, $X_2 = 3$

16/8/23

DUAL SIMPLEX METHOD

Q Min $Z = 2x_1 + x_2$ s.t. $3x_1 + x_2 \geq 3$, $4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \geq 3$, $x_1, x_2 \geq 0$.

Soln Max $Z' = -2x_1 - x_2$ s.t. $-3x_1 - x_2 \leq -3$,
$-4x_1 - 3x_2 \leq -6$, $-x_1 - 2x_2 \leq -3$

Std. form → Max $Z' = -2x_1 - x_2 + 0s_1 + 0s_2 + 0s_3$.
s.t. $-3x_1 - x_2 + s_1 = -3$
$-4x_1 - 3x_2 + s_2 = -6$
$-x_1 - 2x_2 + s_3 = -3$

C_B	B.V	X_B	x_1	x_2	s_1	s_2	s_3	Min
	$C_j \to$		-2	-1	0	0	0	
0	s_1	-3	-3	-1	1	0	0	
0	s_2	-6	-4	$\boxed{-3}$	0	1	0	→
0	s_3	-3	-1	-2	0	0	1	
Z=0,	$Z_j \to$		0	0	0	0	0	
	$\Delta_j \to$		-2	-1	0	0	0	

Here all Δ_j's & X_B's are -ve. ⇒ Solⁿ is non-feasible
so we'll make X_B's +ve or 0.

i) Take most -ve element of X_B as outgoing.
ii) ÷ Δ_j's by corresponding outgoing Row (Take only -ve's)
iii) Choose min of all Δ_j's.
iv) ~~Min~~ Element corresponding to min Δ_j & outgoing one is key element. (intersection)

$-2/-4$ $\quad$ $(-1/-3)$ → min.

C_B	B.V	X_B	x_1	x_2	s_1	s_2	s_3	Min
0	s_1	-1	$\boxed{-5/3}$	0	1	-1/3	0	→
-1	x_2	2	4/3	1	0	-1/3	0	
0	s_3	1	5/3	0	0	-2/3	1	
Z=-2	$Z_j \to$		-4/3	-1	0	1/3	0	
	$\Delta_j \to$		~~-10/3~~ -2/3	0	0	-1/2	0	
$\frac{\Delta_j}{x_1}$			$\frac{-2/3}{-5/3}$	—	—	$\frac{-1/3}{-1/3}$	0	
-2	x_1	3/5	1	0	-3/5	1/5	0	
-1	x_2	6/5	0	1	4/5	-3/5	0	
0	s_3	0	0	0	1	-1	1	
z=								

Q2 Min $z = x_1 + 2x_2$ s.t. $2x_1 + x_2 \geq 4$, $x_1 + 2x_2 \geq 7$.
$x_1, x_2 \geq 0$

Solⁿ Max $z' = -x_1 - 2x_2 + 0s_1 + 0s_2$
s.t. $-2x_1 - x_2 + s_1 = -4$
$-x_1 - 2x_2 + s_2 = -7$

C_B	B.V.	X_B	x_1	x_2	s_1	s_2	Min
	C_j		-1	-2	0	0	
0	s_1	-4	-2	-1	1	0	
0	s_2	-7	$\boxed{-1}$	-2	0	1	→
Z'=0	$Z_j \to$		0	0	0	0	
	$\Delta_j \to$		-1	-2	0	0	
	Δ_j/s_2		1	1	-	—	
			↑				
0	s_1	10	0	3	1	-2	
-1	x_2	7	1	2	0	-1	
Z'=-7	$Z_j \to$		-1	-2	0	1	
	$\Delta_j \to$		0	0	0	-1	

⇒ $x_1 = 0$, $x_2 = 7$ $\quad$ Max $z' = -7$
⇒ Min $z = 7$

Q3 Max $z = -4x_1 - 6x_2 - 18x_3$ s.t. $x_1 + 3x_3 \geq 3$,
$x_2 + 2x_3 \geq 5$ and $x_1, x_2, x_3 \geq 0$.

Solⁿ Max $z = -4x_1 - 6x_2 - 18x_3 + 0s_1 + 0s_2$
s.t. $-x_1 - 3x_3 + s_1 = -3$
$-x_2 - 2x_3 + s_2 = -5$

C_B	B.V.	X_B	x_1	x_2	x_3	s_1	s_2	Min
	C_j		-4	-6	-18	0	0	
0	s_1	-3	-1	0	-3	1	0	
0	s_2	-5	0	$\boxed{-1}$	-2	0	1	→
Z=0	$Z_j \to$		0	0	0	0	0	
	$\Delta_j \to$		-4	-6	-18	0	0	{min 6, 9
	Δ_j/s_2		—	6 ↑	9	-	—	

			-4	-6	-18	0	0	
C_B	B.V.	X_B	x_1	x_2	x_3	S_1	S_2	Min
0	S_1	-3	-1	0	$\boxed{-3}$	1	0	⟶
-6	x_2	5	0	1	2	0	-1	
Z = -30		$z_j \to$	0	-6	-12	0	6	
		$\Delta_j \to$	-4	0	-6	0	-6	
		Δ_j / S_1	4	–	2 ↑	–	–	min{2,4}
-18	x_3	1	1/3	0	1	-1/3	0	
-6	x_2	3	-2/3	1	0	2/3	-1	
Z = -36		$z_j \to$	-2	-6	-18	2	6	
		$\Delta_j \to$	-2	0	0	-2	-6	

Max $Z = -36$, $x_1 = 0$, $x_2 = 3$, $x_3 = 1$

Q4 Min $Z = 6x_1 + x_2$ s.t. $2x_1 + x_2 \geq 3$, $x_1 - x_2 \geq 0$

Sol Max $z' = -6x_1 - x_2 + 0S_1 + 0S_2$

s.t. $-2x_1 - x_2 + S_1 = -3$

$-x_1 + x_2 + S_2 = 0$

		c_j	-6	-1	0	0	
C_B	B.V.	X_B	x_1	x_2	S_1	S_2	Min
0	S_1	-3	-2	$\boxed{-1}$	1	0	⟶
0	S_2	0	-1	1	0	1	
Z' = 0		$z_j \to$	0	0	0	0	
		$\Delta_j \to$	-6	-1	0	0	
		Δ_j / S_1	3	1 ↑	–	–	min{3,1}
-1	x_2	3	2	1	-1	0	
0	S_2	-3	$\boxed{-3}$	0	1	1	⟶
z' = -3		$z_j \to$	-2	-1	1	0	
		$\Delta_j \to$	-4	0	-1	0	
		Δ_j / S_2	4/3 ↑	–	–	–	

$-1 + \frac{2}{3}$ $2 + 6$

-1	X_2	1	0	1	-1/3	2/3
-6	X_1	1	1	0	-1/3	-1/3
z' = -7		$z_j \to$	-6	-1	7/3	8/3
		$\Delta_j \to$	0	0	-7/3	-8/3

Max $z' = -7$ → Main $z = 7$, $x_1 = x_2 = 1$

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UNIT- II

ASSIGNMENT PROBLEM

Suppose there are n jobs to be performed & n persons are available for doing these jobs. Assume that each person can do each job at a time though with varying degree of efficiency.

let c_{ij} be the cost (payment). If the ith person is assigned the jth job. The problem is to find the assignment so that total cost for performing all jobs is minimum.

$$\begin{array}{cc|ccccccc} & & & & & \text{jobs} & & & \\ & & 1 & 2 & 3 & \cdots & j & \cdots & n \\ \hline & 1 & c_{11} & c_{12} & c_{13} & \cdots & c_{1j} & \cdots & c_{1n} \\ \text{persons} & 2 & c_{21} & c_{22} & c_{23} & \cdots & c_{2j} & \cdots & c_{2n} \\ & 3 & c_{31} & \vdots & \vdots & & & & \\ & \vdots & \vdots & \vdots & \vdots & & & & \\ & i & c_{i1} & c_{i2} & c_{i3} & \cdots & & & \\ & \vdots & \vdots & \vdots & \vdots & & & & \\ & n & c_{n1} & c_{n2} & c_{n3} & \cdots & & & \end{array}$$

Ex: Assigning classes to rooms,
" drivers " trucks,
" men to jobs.

Mathematical Formulation of Assignment Problem

Mathematically the assignment problem can be stated as

Minimize the total cost $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$.

subject to the restrictions of the form

$$x_{ij} = \begin{cases} 1, & \text{if ith person is assigned to jth job.} \\ 0, & \text{Otherwise.} \end{cases}$$

$\sum_{j=1}^{n} x_{ij} = 1$. (1 job is done by the ith person; $i = 1, \dots, n$)

$\sum_{i=1}^{n} x_{ij} = 1$ (only 1 person should be assigned the jth job ~~1 job is done by the ith person~~)

HUNGARIAN METHOD (to solve assignment problem)

Step 1 → a) If the no of sources = no of destinations. Then, go to step 3.

b) If no of sources ≠ no of destinations. Then, go to ~~st~~ step 2.

Step 2 → Add a dummy source or dummy destination so that the cost table becomes a square matrix. The cost entries of dummy sources/destinations are always zero.

Step 3 → find out the smallest element in each row of the given matrix & then subtract the same from each column of that row.

Step 4 → find out the smallest element in each column & then subtract the same from each element of that row.

Step 5 → In the modified matrix of step 4 do the optimal assignment.

i) Examine the rows until a row with single zero is obtained. Mark this zero as ($\boxed{0}$) and cross out (X) all other zeroes in that column. Continue in this manner until all the rows have been considered.

ii) Repeat this procedure for each column of the reduced matrix.

iii) If a row or column has two or more zeros & 1 can't be assigned then assign arbitrary any one of these zeroes and cross out all other zeroes of that row/column.

iv) Repeat 1st point to 3rd point successively until the chain of assigning complete.

Step 6 → If the no of assignments = n, then an optimal sol^n is obtained.

Q. Solve the minimization assignment problem

Persons

		I	II	III	IV
Tasks	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

Sol^n Step 1 → It is a balanced assignment (∵ no of tasks = no of persons).

Step 3 → Subtract the smallest element of each row from each element of that row.

First reduced matrix

Persons

$$\begin{array}{cccccc} & & \text{I} & \text{II} & \text{III} & \text{IV} \\ \text{Tasks} & A & 0 & 18 & 9 & 3 \\ & B & 9 & 24 & 0 & 22 \\ & C & 23 & 4 & 3 & 0 \\ & D & 9 & 16 & 14 & 0 \end{array}$$

Step4 → Subtract smallest from each column.

2nd reduced matrix

Persons

$$\begin{array}{cccccc} & & \text{I} & \text{II} & \text{III} & \text{IV} \\ \text{Tasks} & A & \boxed{0} & 14 & 9 & 3 \\ & B & 9 & 20 & \boxed{0} & 22 \\ & C & 23 & \boxed{0} & 3 & \cancel{0} \\ & D & 9 & 12 & 14 & \boxed{0} \end{array}$$

Step6 → here no of marked zeroes (□) = no of tasks = no of persons = 4.

So, optimumal assignment has been attained.

Now, for minimum cost :-

A − I 8
B − III 4
C − II 19
D − IV 10
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Step7 [Optimality criteria].

a) If all zero elements in the ~~x~~ cost matrix either marked □ or are crossed (x) and there is exactly one assignment in each row and column, then it is optimal sol^n^.

If total cost associated with this sol^n^ is obtained by adding the original cost elements in the occupied cells.

b) If a zero element in a row or column were chosen arbitrarily for assignment in step ⑤, ∃ an alternate optimal sol^n^.

c) If there is no assignment in a row (or column) then ⇒ total no of assignments are less than the no of rows/columns in a square matrix.

Step8 [Revise the opportunity cost matrix]

Draw a set of horizontal & vertical lines to cover all the zeroes in the revised cost matrix obtained from the step ⑤ by using the following procedure :-

a) For each row in which no assignment are made mark a tick (✓).

b) Examine the marked row, if any zero element is present in these rows mark a tick (✓) to the respective columns containing zeroes.

c) Examine marked columns, if any assigned zero element is present in these columns tick (✓) the respective rows containing assigned zeroes.

d) Repeat this process until no more rows or columns can be marked.

e) Draw a straight line through each marked column & unmarked row.

If the no of lines drawn = no of rows/columns then, the current sol^n^ is optimal sol^n^ Otherwise go to step ⑨.

Step 9 [Develop a new revised opportunity cost matrix

a) Among the elements in the matrix not covered by any line choose the smallest element & call this value 'K'.

b) Subtract K from every element in the matrix ~~that~~ is not covered by a line.

c) Add K to every element in the matrix covered by 2 lines i.e. intersection of 2 lines.

d) Element in the matrix covered by 1 line remains unchanged.

Step 10 Repeat step 5-9 until an optimal solⁿ is obtained.

Q. Program →

Programmer ↓

	A	B	C
1	120	100	80
2	80	90	110
3	110	140	120

Solⁿ

	A	B	C
1	40	20	0
2	0	10	30
3	0	30	10

2nd s

	A	B	C
1	40	10	0
2	0	0	30
3	0	20	10

optimal solⁿ obtained

1	A	80
2	B	90
3	C	110
		280

Q

	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

line ↓ unticked row & ticked column

Solⁿ 1st

	I ✓	II	III	IV ✓	V	
A	5	0	8	10	11	
B	0	6	15	10	3	✓
C	8	5	0	0	0	
D	0	4	2	0	5	✓
E	3	5	6	0	8	✓

2nd

	I	II	III	IV	V
A	3	0	4	11	10
B	0	2	9	9	0
C	3	0	0	0	0
D	0	4	0	3	6
E	0	2	1	0	6

	I	II	III	IV	V
A	7	0	8	12	11
B	0	4	13	10	1
C	10	5	0	2	0
D	0	2	0	0	3
E	3	3	2	0	6

optimal.

A → II	5
B → I	3
C → V	2
D → III	9
E → IV	4
+	23

Q

	P	Q	R	S	T
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Sol

1st

	P	Q	R	S	T
A	20	10	0	60	10
B	24	12	0	66	12
C	18	9	0	57	12
D	20	12	0	60	12
E	20	8	0	56	12

2nd

	P	Q	R✓	S	T	
A	2	2	$\cancel{0}$	4	$\boxed{0}$	
B	6	4	$\boxed{0}$	10	2	✓
C	$\boxed{0}$	1	$\cancel{0}$	1	2	
D	2	4	$\cancel{0}$	4	2	✓
E	2	$\boxed{0}$	$\cancel{0}$	$\boxed{\cancel{0}}$	2	

3rd

	P	Q	R	S	T	
A	2	2	2	4	$\boxed{0}$	
B	4	2	$\boxed{0}$	8	$\cancel{0}$	X
C	$\boxed{0}$	1	2	1	2	
D	$\cancel{0}$	2	$\cancel{0}$	2	$\boxed{0}$	
E	2	$\boxed{0}$	2	$\cancel{0}$	2	

3rd

	P✓	Q	R✓	S	T✓	
A	2	2	2	4	$\boxed{0}$	✓
B	4	2	$\boxed{0}$	8	$\cancel{0}$	✓
C	$\boxed{0}$	1	2	1	2	✓
D	$\cancel{0}$	2	$\cancel{0}$	2	$\cancel{0}$	✓
E	2	$\boxed{0}$	2	$\cancel{0}$	2	

4th →

	P✓	Q	R✓	S	T✓	
A	2	1	2	3	$\boxed{0}$	✓
B	4	1	$\boxed{0}$	7	$\cancel{0}$	✓
C	$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$	2	✓
D	$\cancel{0}$	1	$\cancel{0}$	1	$\boxed{\cancel{0}}$	✓
E	3	$\boxed{0}$	3	$\cancel{0}$	3	

5th

	P	Q	R	S	T
A	2	$\boxed{0}$	2	2	$\cancel{0}$
B	4	$\cancel{0}$	$\boxed{0}$	6	$\cancel{0}$
C	0	-1	2	-1	2
D	0	0	0	0	0
E	4	0	4	0	4

~~6th~~ 4th

	P	Q	R	S	T
A	2	1	2	3	$\boxed{0}$
B	4	1	$\boxed{0}$	7	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$	2
D	$\boxed{0}$	1	2 $\cancel{0}$	1	$\cancel{0}$
E	3	$\cancel{0}$	3	$\boxed{0}$	3

A → T → 75
B → R → 66
C → Q → 66
D → P → 80
E → S → 112

399

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VARIATIONS OF THE ASSIGNMENT PROBLEM

★ **Multiple Optimal Solⁿ**

While making an assignment in the redu assignment matrix it is possible to have

2 or more ~~to~~ ways to strike off a certain no. zeroes.
Such a situation indicates that there are multiple ~~m~~ optimal sol^n with the same optimal value of the objective fun^n.

* Maximization Case in Assignment Problem

If instead of cost matrix, a profit matrix is given then assignments are made in such a way that total profit is maximized.
The profit maximization problems are solved by converting them into a cost minimization problem in either of the following 2 ways :-

① Put a -ve sign before each of the element in the profit matrix, in order to convert profit value into a cost value.

② Locate the largest element in the profit matrix & subtract all the elements of the matrix from the largest element including itself.

Q Maximize the sales →

		A	B	C	D	E
	1	32	38	40	28	40
	2	40	24	28	21	36
sales man →	3	41	27	33	30	37
	4	22	38	41	36	36
	5	29	33	40	35 ~~38~~	39

1st

	A	B	C	D	E
1	9	14	9	21	1
2	1	25	21	28	13
3	0	22	16	19	12
4	27	11	8	13	13
5	20	16	9	1	10

1st

	A	B	C	D	E
1	9	3	1	13	1
2	1	17	13	20	5
3	0	14	8	11	4
4	19	3	0	5	5
5	12	8	1	6	2

2nd

	A	B	C	D	E
1	8	2	0	12	0
2	0	16	12	19	4
3	0	14	8	11	4
4	19	3	0	5	5
5	11	7	0	5	1

3rd

	A ✓	B	C	D	E	
1	8	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$	
2	$\boxed{0}$	14	12	14	4	✓
3	$\cancel{0}$	12	8	6	4	✓
4	19	1	$\boxed{0}$	$\cancel{0}$	5	
5	11	5	$\cancel{0}$	$\boxed{0}$	1	

4th

	A	B	C	D	E
1	12	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$
2	$\boxed{0}$	10	8	10	$\cancel{0}$
3	$\cancel{0}$	8	4	2	$\boxed{0}$
4	23	1	$\boxed{0}$	$\cancel{0}$	5
5	15	5	$\cancel{0}$	$\boxed{0}$	1

1→B → ~~28~~
2→A → 40
3→E → 37
4→C → 41
5→D → 35
~~188~~ 191

4th again

	A	B	C	D	E
1	12	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$
2	$\boxed{0}$	10	8	10	$\cancel{0}$
3	$\cancel{0}$	8	4	2	$\boxed{0}$
4	23	1	$\cancel{0}$	$\boxed{0}$	5
5	15	5	$\boxed{0}$	$\cancel{0}$	1

1→B → 38
2→A → 40
3→E → 27
4→D → 36
5→C → 40
191

⇒ Multiple optimal solⁿ.

Unbalanced ASSIGNMENT Problem

The hungarian method for solving assignment problem requires that no of rows and columns in assignment matrix should be equal.

However, when the cost matrix is not a square matrix. The assignment problem is called unbalanced problem.

In such cases before applying hungarian method dummy rows or columns are added in the matrix (with zeroes as a cost element) in order to make it a square matrix.

Restrictions on Assignment

Sometimes it may so happen that a particular resource (say a man or a machine) can't be

assigned to a particular activity (or Job). In such cases the cost of performing that particular activity by a particular resource is considered to be very large (written as 'M' or '∞') so as to prohibit the entry of dispair of resource activity into the final solⁿ.

Q.

Location ~~machine~~ →

		A ~~Location~~	B	C	D	E
Machine	M1	9	11	15	10	11
	M2	12	9	∞	10	9
	M3	∞	11	14	11	7
	M4	14	8	12	7	8
	M5	0	0	0	0	0

1st

	A	B	C	D	E
M1	$\boxed{0}$	2	6	1	2
M2	3	$\boxed{0}$	∞	1	$\cancel{0}$
M3	∞	~~8~~4	7	3	$\boxed{0}$
M4	7	~~8~~1	5	$\boxed{0}$	1
M5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

M1→A → 9
M2→B → 9
M3→E → 7
M4→D → 7
M5→C → 0
32

TRANSPORT~~-EQ~~ATION PROBLEM

The structure of transportation problem involves a large no. of shipping routes from several supply centres to several demand centres.

Thus, objective is to determine shipping routes betn supply centres & demand centres in order to satisfy the required quantity of goods or services at each destination centres with available quantity of goods or services at each supply centres at minimum transportation cost and/or time.

☆ Mathematical Model of transportation Problem:

Q A Company has 3 production facilities S_1, S_2, S_3 with production capacity of 7, 9, 18 units resp. per week of a product resp. These units are to be shipped to 4 warehouses D_1, D_2, D_3, D_4 with requirement of 5, 8, 7, 14 units per week resp. The transportation cost (in rupees) per unit betn factories to warehouses are given in table below:

	D_1	D_2	D_3	D_4	supply
S_1	$9^{x_{11}}$	$30^{=x_{12}}$	$50^{=x_{13}}$	$10^{=x_{14}}$	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

→ balanced q/c total supply = Demand = 34 i.e. same.

formulate this transportation problem as LP model to minimize the total cost.

Soln let x_{ij} = no. of units of product to be transported from facility $i (i=1,2,3)$ to warehouse $j (j=1,2,3,4)$.

The transportation problem is stated as LP model as follows:

Min (Total transportation cost) $Z = 19x_{11} + 30x_{12} + 50x_{13} + 10x_{14} + 70x_{21} + 30x_{22} + 40x_{23} + 60x_{24} + 40x_{31} + 8x_{32} + 70x_{33} + 20x_{34}$

Subject to the Constraints

$$\left.\begin{aligned} x_{11}+x_{12}+x_{13}+x_{14} &\le 7 \\ x_{21}+x_{22}+x_{23}+x_{24} &\le 9 \\ x_{31}+x_{32}+x_{33}+x_{34} &\le 18 \end{aligned}\right\} \rightarrow \text{supply}$$

and

$$\left.\begin{aligned} x_{11}+x_{21}+x_{31} &= 5 \\ x_{12}+x_{22}+x_{32} &= 8 \\ x_{13}+x_{23}+x_{33} &= 7 \\ x_{14}+x_{24}+x_{34} &= 14 \end{aligned}\right\} \rightarrow \text{Demand}$$

→ In the above LP model; $x_{ij} \ge 0$ for $i=1,2,3$ and $j=1,2,3,4$.

Also there are $m \times n =$ ~~3×12~~ $3 \times 4 = 12$ decision variables x_{ij} and $m+n = 7$ constants where m is the no. of rows & n is no. of columns in the General transportation Problem.

☆ General Mathematical Model of transportation Problem

let there be m sources of supply $S_1, S_2, \cdots, S_m$ having $a_i (i=1,2,\cdots,m)$ having units of supply respectively to be transported to n destinations $D_1, D_2, \cdots, D_n$ with $b_j (j=1,2,\cdots,n)$ units of demand resp.

Let c_{ij} be the cost of shipping one unit of the product from source i to destination j & x_{ij} represents no. of units shipped from

source i to destination j, the problem is to determine the transportation schedule so as to minimize total transportation cost while satisfying supply & demand cond^ns.
Mathematically the transportation problem in general may be stated as follows:

$$\text{Min(Total transportation cost)} Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to constraints, $\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \cdots, m$
for supply.

for demand, $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, \cdots, n$

and $x_{ij} \geq 0$ $\forall i, j$.

★ Existence of feasible sol^n

A necessary & suff. cond^n for a feasible sol^n to the transportation eq^n is:
total supply = total demand.

i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (also called RIM cond^n)

	D_1	D_2	$\cdots$	D_n	Supply
S_1	$\textcircled{x_{11}} C_{11}$	$\textcircled{x_{12}} C_{12}$	$\longrightarrow$	$\textcircled{x_{1n}} C_{1n}$	a_1
S_2	$\textcircled{x_{21}} C_{21}$	$\textcircled{x_{22}} C_{22}$	$\longrightarrow$	$\textcircled{x_{2n}} C_{2n}$	a_2
$\vdots$	$\vdots$	$\vdots$	$\longrightarrow$		$\vdots$
S_m	$\textcircled{x_{m1}} C_{m1}$	$\textcircled{x_{m2}} C_{m2}$	$\longrightarrow$	$\textcircled{x_{mn}} C_{mn}$	a_m
Demand	b_1	b_2	$\cdots$	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

In this problem there are $m+n$ ~~constants~~ Constraints, 1 for each source of supply & destination and $m \times n$ variables.
Since all $m+n$ ~~constants~~ Constraints are eq^ns ∴ one of these eq^ns is extra.
The extra ~~constant~~ Constraint (can be derived from other constant without affecting the feasible sol^n)
It follows that any feasible sol^n for a transport ~~eq^n~~ problem must have $m+n-1$ non-ve basic variable (or allocations) x_{ij} satisfying the RIM cond^ns.

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Remarks → ① When total supply = total demand, the problem is called balanced transportation problem otherwise unbalanced " ".
The unbalanced " " can be made balanced by adding dummy supply centre (row) or a dummy demand centre (column) as the need arises.

② When no of +ve allocations (values of decision variables) at any stage of the feasible sol^n is less than the required no's (rows + columns − 1)$(m+n-1)$ i.e. no of independent constant eq^ns, the sol^n is s.t.b. Degenerate. Otherwise non-degenerate.

③ Cell in a transportation table having +ve allocation i.e. $x_{ij} > 0$ are called Occupied cell Otherwise are known non-occupied.

Q

	D_1	D_2	D_3	D_4	Supply
S_1	2 ⑥	3	11	7	6
S_2	1 ①	0	6	1	1
S_3	5	8 ⑤	15 ③	9 ②	10
Demand	7	5	3	2	17

Here transportation problem is balanced
But no of allocation = 5
and $m+n-1$ = 6
i.e. both unequal.
⇒ sol^n is degenerate.

Q

	D1	D2	D3	D4	Supply
S1	19⑤	30②	50	10	7
S2	70	30⑥	40③	60	9
S3	40	8	70④	20⑭	18
Demand	5	8	7	14	34

Total ~~feasible~~ cost = 19x5 → 95
30x2 → 60
30x6 → 180
40x3 → 120
70x4 → 280
20x14 → 280
1015

here m+n-1 = 3+4-1=6
and we have 6 no of costs. so, 1015 is sol of the

LEAST COST METHOD

Q1

	D1	D2	D3	D4	Supply
S1	19	20	50	10⑦	7
S2	70②	30	40⑦	60	9
S3	40③	8⑧	70	20⑦	18
Demand	5	8	7	14	34

No of allocations = m+n-1 so, Non-degenerate.
Total cost → 140
120
64
280
70
140
814

Q2

	D1	D2	D3	D4	Supply
S1	2⑥	3	11	7	6
S2	1	0①	6	1	1
S3	5①	8④	15②	9②	10
Demand	7	5	3	2	17

No of allocations = m+n-1 so, non-degenerate.
Total cost → 2x6 + 5x1 + 0x1 + 8x4 + 15x3 + 9x2
→ 12+5+32+45+18 = 112

VOGEL ~~Me~~ APPROXIMATION METHOD

4th row diff		D1	D2	D3	D4	Supply	1st Row diff	2nd	3rd
40	S1	19⑤	30	50	10②	7	9	9	40
20	S2	70	30	40⑦	60②	9	10	20	20
—	S3	40	8⑧	70	20⑩	18	12	20	50 Max
	Demand	5	8	7	14	34			
	1st column diff	21	22	10	10				
	2nd column diff	21 Max.	—	10	10				
	3rd column diff	—	—	10	10				
	4th column diff	—	—	10	50 Max				

Nondegenerate problem.
Total cost → 19x5 + 8x8 + 40x7 + 10x2 + 60x2 + 20x10
→ 95+64+280 + 20+120 + 200 = 779

Test for Optimality

Once an initial sol^n is obtained. The next step is to check its optimality in terms of feasibility of the sol^n and total minimum transportation cost. The test of optimality begins by calculating an opportunity cost associated with each unoccupied cell in the transportation ~~cell~~ table.

An unoccupied cell with the largest -ve opportunity cost is selected to include in the new set of transportation allocations. This value indicates per unit cost deduction that can be achieved by making appropriate allocation in the unoccupied cell. This cell is known as an Incoming cell (or Variable).

The outgoing cell from the current sol^n is the occupied cell (basic Variable) where allocation will become zero as allocation is made in the unoccupied cell with the largest -ve opportunity cost.

Such as exchange reduces the total transportation cost. The process is continued until no -ve opportunity cost is left. i.e. the current sol^n is optimal sol^n.

The modified distribution (MODI) Method also called u-v method OR Method of Multiplier) is used to calculate opportunity cost associated with each unoccupied cell and then improving the current sol^n leading to an optimal sol^n.

* Steps of MODI Method

The steps to evaluate the unoccupied cells are as follows:-

Step1> For an initial basic feasible sol^n with $(m+n-1)$ occupied cells. Calculate u_i & v_j for rows & columns. The initial sol^n can be obtained by any of the three methods done earlier.

To start with any one of u_i & v_j is assigned the value 0. It's better to assign 0 to a particular u_i or v_j where there are max. no of allocations in a row or a column resp., as this will reduce the considerable arithmetic work. The values of u_i & v_j's for other rows & columns is calculated by using the relationship

$c_{ij} = u_i + v_j$ for all occupied cell (i,j)

Step2 For unoccupied cell, calculate the opportunity cost by using relationship

$d_{ij} = c_{ij} - \cancel{d}(u_i + v_j) \quad \forall i$ and j.

Step3 Examine sign of each d_{ij}

i) if $d_{ij} > 0$, then current basic feasible sol^n is optimal.

ii) if $d_{ij} = 0$, then current basic feasible sol^n will remain unaffected but an alternate sol^n exist.

iii) if one or a more $d_{ij} < 0$ then an improved sol^n can be obtained by entering an unoccupied cell (i,j) into the sol^n basis (table).

An occupied cell having the largest -ve value of d_{ij} is selected for entering into the sol^n basis (table)

step4 Construct a closed ~~box~~ path (Loop) for the unoccupied cell with the largest -ve value of d_{ij}. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in the cell. Trace a path along the rows or columns to an occupied cell, mark the corners with a minus sign (−) & continue down the column (or row) to an occupied cell.

Then mark the corner with plus sign (+) & the minus sign (−) alternately. Close the path back to the unoccupied cell.

step5 Select the smallest quantity amongst the cells marked with minus sign(−) on the corners of closed loop. Allocate this value to the selected unoccupied cell & add it to occupied cells marked with plus sign (+) & subtract it from the occupied cells marked with (−) sign.

step6 Obtain a new improved sol^n by allocating units to the unoccupied cell acc. to step5 & calculate the new total transportation cost.

step7 Test optimality of the revised sol^n. The procedure terminates when all $d_{ij} \geq 0$ for unoccupied cells.

Vogel's Approximation Method

This method is preferred over NWCR & LCM methods. In this ethod, an allocation is made on the basis the opportunity (or penalty or extra) cost at would have been incurred if the allocation contains cells with min unit transp. cost re missed. Hence, the allocations are made uch a way that the penalty cost is inimized. An initial soln obtained by using is method is nearer to an optimal soln or the optimal soln itself. The steps are:-

p1 Calculate the penalties for each row (column) taking the difference b/w the smallest & next smallest unit transportation cost in same row (column). This diff indicates the nalty or extra cost that has to be paid decision-making fails to allocate to the ll with the minimum unit transportation cost.

p2 Select the row or column with the argest penalty & allocate as much as possible the cell that has the least cost in the lected row or column that satisfies the im conditions. If there is a tie in the values penalties, it can be broken by selecting the ll where the max allocation can be made.

p3 Adjust the supply & demand and cross t the satisfied row or column. If a row & column are satisfied simultaneously, only one f them is crossed out & the remaining row column) is assigned a zero supply (demand). Any row or column with zero supply or demand should not be used in computing future penalties.

Step 4 Repeat steps 1 to 3 until the available supply at various sources & demand at various destinations is satisfied.

Remark 1. The closed loop (path) starts & ends at the selected unoccupied cell. It consists of successive horizontal & vertical (connected) lines whose end points must be occupied cells, except an end point associated with entering unoccupied cell. This means that every corner element of the loop must be an occupied cell.

It is immaterial whether the loop is traced in a clockwise or anti-clockwise dirn & whether it starts up down, right or left (but never diagonally). However for a given sol only one loop can be constructed for each unoccupied cell.

2. There can only be one plus ~~(+)~~ (+) sign & only one (-) sign in any given row or column.

3. The closed path indicates changes involved in reallocating the shipments

Close-loop in Transportation Table and its Properties :-

Any basic feasible soln must contain $m+n-1$ non zero allocations provided.

(i) any two adjacent cells of the ordered set lie either in the same row or in the same column.

(ii) no three or more adjacent cells in the ordered set lie in the same row or column. The first cell of the set must follow the last in the set, ie each cell (except the last) must appear only once in the ordered set

Remarks

(1) every loop has an even no. of cells & has atleast four cells.

(2) The allocations are said to be in independent position if it is not possible to increase or decrease any individual allocation without changing the posns of these allocations, or if a closed loop can't be formed through these allocations without violating the rim conditions.

(3) each row and column in the transportation table should have only one plus & minus sign. All cells that have a plus or a minus sign, except the starting unoccupied cell, must be occupied cells.

(4) Closed loops may ~~be~~ or may not be in the shape of a square.

Degeneracy in the Transportation prob

when no. of allocations in the basic feasible soln is less than $m+n-1$ then the problem of degenera occurs.

Degeneracy problem can occur in two w

1. Basic feasible soln may be degen- from the initial stage onwards.
2. Degeneracy may occur during the optimality test.

Visualation of the degeneracy during the initial stage.

1→ The extremely small quantity denoted by δ or ε is introduced in the le cost independent cell subject to the following assumptions If necessary ~~to~~ two or more deltas can be introduced in the least and second least cost ind- endent cell.

(i) $\Delta + 0 = \Delta$ (ii) $\Delta - 0 = \Delta$

(iii) $x_{ij} + \Delta = x_{ij} - \Delta = x_{ij}$

- If there are more than one deltas in the soln $\Delta < \Delta'$ whenever Δ is ab Δ'.

If Δ and Δ' are in the same row $\Delta < \Delta'$ when Δ is to the left of Δ'

Assig Pg T348 → Q2(i), Q3, Q7, Q9
Pg AP 32 Q1(ii), (viii), Q5, Q7, Q11, Q17,

Q1. Find the optimal solⁿ for

	X	Y	Z	
A	8	7	60 3	60
B	50 3	8	20 9	70
C	11	80 3	5	80
	50	80	80	

Here $\Sigma a_i = 210 = \Sigma b_j$
So, It is a balanced transpoitation problem

Using Vogel's Approximation Method, initial basic feasible solⁿ is

Here no. of allocations = 4
$m+n-1 = 3+3-1 = 5$
$4 < 5 \; (m+n-1)$

So, degeneracy occurs.

To find u_i & v_j

			u_i
		3	3
3		9	9
	3	5	5
-6	-2	0	v_j

To find d_{ij}

11	6	
	1	
12		

Here all $d_{ij} > 0$

~~current solⁿ is~~
⇒ optimal solⁿ.

opt cost = $60 \times 3 + 50 \times 3 + 20 \times 9 + 10 \times 3 + 1 \times 5$
= 750

Q2. Solve the Trans prob to find opt solⁿ

	X	Y	Z	
I	5(2)	7	4	5
II	3	3	8(1)	8
III	5	7(4)	7	7
IV	2(1)	2(6)	10(2)	14
	7	9	18	

$\Sigma a_i = 34 = \Sigma b_j$ balanced TP.

By VAM, no. of all. = 6 = 6 (m+n-
non degenerate.

To find u_i & v_j

			u_i
2			1
		1	-1
	4		-2
1	6	2	0
1	6	2	v_j

d_{ij} =

	0	1
3	-2	
6		7

Here all d_{ij} is not +ve
current solⁿ is not optimal.

5(2)	7	4	5
3	+θ(3)	−θ 8(1)	8
5	7(4)	7	7
2(1)	−θ 2(6)	10(2) +θ	14
7	9	18	

~~Min (8−θ),~~ = Min(8−θ, 2−θ) = 0,
2−θ = 0

$\boxed{\theta = 2}$

Now New sol^n is

	X	Y	Z	
I	5(2)	7	4	5
II	3	2(3)	6(1)	8
III	5	7(4)	7	7
IV	2(1)	6	12(2)	14
	7	9	18	

ui & vj

				ui
	2			1
		3	1	−1
		4		0
	1		2	0
vj	1	4	2	

opt sol^n

dij

	2	1
3		
4		5
	2	

all dij > 0

opt cost = 768. A

Pg TP/11

Exercise

① (i)

6⑥	4⑧	1	5	14
8	9②	2⑭	7	16
4	3	6①	2④	5
6	10	15	4	

(ii)

13(2000)	11	15	20	2000
17(1000)	14(3000)	12(2000)	13	6000
18	18	15(2000)	12(5000)	7000
3000	3000	4000	5000	

(iii)

9③	12②	9	8	4	3	5
7	3②	6⑤	8①	9	4	8
4	5	6	8⑥	10	14	6
7	3	5	7	10⑥	9①	7
2	3	8	10	2	4③	3
3	4	5	7	6	4	

② (i)

50	30①	220	1
90③	45①	110	4
250①	200	50③	4
4	2	3	

(ii)

6	3⑫	5	4⑨	22
5	9	2⑮	7	15
5	7	8	6	8
7	12	17	9	

6	3⑫	5①	4⑨	22
5	9	2⑮	7	15
5⑦	7	8①	6	8
7	12	17	9	

Pg TP/20. By VAM

4②

2 ⑤	7	4	5	2	2	5	—
3	3	1 ⑧	8	2	—	—	—
5	4 ⑦	7	7	1	1	1	1
1 ②	6 ②	2 ⑩	14	1	1	5	5
7	9	18					

1	1	1
1	2	2
1	2	–
4	2	–

no. of allocations = 6.
non degenerate

ui&vj

			ui
2			1
		1	-1
	4		-2
1	6	2	0
vj 1	6	2	

dij

	0	1
3	-2	
6		7

all dij are not +ve

current solⁿ is not optimal

2 ⑤	7	4	5
3	$+\theta$ 3	$-\theta$ 1 ⑧	8
5	4 ⑦	7	7
1 ②	$-\theta$ 6 ②	$+\theta$ 2 ⑩	14
7	9	18	

$\min(8-\theta,\ 2-\theta) = 0$

$2-\theta = 0 \Rightarrow \theta = 2$

New solⁿ =

2 ⑤	7	4	5
3	3 ②	1 ⑥	8
5	4 ⑦	7	7
1 ②	6	2 ⑫	14
7	9	18	

ui&vj

			ui
2			1
	3	1	-1
	4		0
1		2	0
vj 1	4	2	

dij

	2	1
3		
4		5
	2	

all dij ≥ 0

current solⁿ is optimal

eg ③

1 ㉚	2	1 ⑩	4	30	0	0	1	1
3	3 ⑳	2 ⑳	1 ⑩	50	1	1	1	1
4	2 ⑳	5	9	20	2	2	3	—
20	40	30	10					
2	0	1	3					
2	0	1	—					
—	0	1	—					
—	1	1	—					

no of allocation = 6.

non-degenerate.

ui & vj

				ui
1		1		-1
	3	2	1	0
	2			-1
vj ~~8~~	3	2	1	

dij

	0		4
1			
3		4	9

all dij >0

eg TP/34 Q5

27 ㉚	23 (70)	31 (50)	69	150	4	4	4	4
40	45	40	72 ㊵	40	22	22	—	—
30 (60)	54	75	57 ⑳	80	5	5	5	5
90	70	50	60					
17	22	4	25					
17	—	4	25					
3	—	4	12					
3	—	4	—					

Min cost = 8190

no. of allocations = 6 = 4+3-1 = 6

non degenerate

ui&vj

				ui
27	23	31.		0
			32	22
30			57	3
vj 27	23	31	54	

69
54
15

di

dij

			15
5	44	31	
	28	1	

all dij >

current optimal solⁿ

Q7

9 ㉚	5	8 (95)	5 (100)	225	0	0	4
9 (75)	10	13	7	75	2	1	1
14 ⑳	5 (80)	3	7	100	2	2	
125	80	95	100				
0	0	5	2				
0	0	—	2				
0	0	—	—				

no. of allocation = 6 = 4+3-1 = 6.

non-degenerate

Min cost 9x30+ 8x95+5x100+9x75+ 14x20+5x80

= 2885

To ui&vj

				ui
9		8	5	9
9				9
14	5			14
Vj 0	-9	-1	-4	

	5		
	10	5	2
		-10	-3

dij < 0.
current solⁿ is not optimal

9 ㊿ +θ	5	8 −θ (95)	5 (100)	225
9 (75)	10	13	7	75
14 ⑳ −θ	5 (80)	3 (θ)	7	100
195	80	95	100	

min { 20-0, 95-0 } = 0
θ=20

9 ㊿	5	8 (75)	5 (100)
9 (75)	10	13	7
14	5 (80)	3 (20)	7

Q11 By VAM

3 ⑮	6	8	5 ⑤	20	2	2	2
6	1 ⑲	2	5 ⑨	28	1	3	1
7	8	3 ⑬	9 ④	17	4	4	2
15	19	13	18				
3	5	1	0				
3	–	1	0				
3	–	–	0				

no of allocation = 6

non-degenerate.

Min cost = 3×15 + 5×5 + 1×19 + 5×9 + 3×13 + 9
= 45 + 25 + 19 + 45 + 39 + 36
= ~~199~~ 209

ui&vj

				ui
3		8	5	5
	1		5	5
		3	9	9
vj -2	-4	-6	0	

dij

	~~4~~ 5	9	~~4~~
~~10~~ 3		3	
0	3		

all dij ≥ 0
current optimal solⁿ

3 ⑮	6	8	5
6	1	2	5
7	8	3	9

5	4.75	4.25	110000	} 900.
5	5.5	6.75	20000	
4.5	6	5	330000	
5.5	6	4.5	440000	
275000	50000	660000	→ 985.	

not Balanced.

Eg TP → 48

Q2(i)

4①	7	3①	8	2②	4	1	1	1	1	1
1⑦	4	7	3	8	7	2	2	–	–	–
7	2③	4⑥	7	7	9	2	2	2	3	–
8	8	2	4②	7	2	2	2	2	2	2
8	3	7	2	2	22.					
3	2	1	1	5						
3	2	1	1	–						
3	5	1	3	–						
3	–	1	3	–						
4	–	1	4	–						

m+n−1 = 4+5−1 = 8

but no of allocations = 7

7 < 8.

degenerate.

Min cost = 4×1 + 1×7 + 2×3 + 3×1 + 4×6 + 4×2 + 2×2

= 4+7+6+3+24+8+4

= 56

ui & vj

					ui
	4		3		2	−1
	1					−4
		2	4			0
				4		
vj	5	2	4		3	

③

55	30⑩	40⑳	50	40⑩	40	10	10
35⑳	30	100	45	60	20	5	5
40⑤	60	95	35㉚	30⑤	40	5	5
25	10	20	30	15			
5	0	55	10	10			
5	0	–	10	10			
5	0	–	–	10			
5	–	–	–	10			
15	–	–	–	10			

⑦

10⑩	20	5	7	10	2	5	10	10	–
13	9⑳	12	8	20	1	3	4	4	
4㉚	15	7	9	30	3	3	11	–	
14	7⑩	1⑳	0⑩	40	1	6	7	7	
3⑳	12㉚	6	9	50	3	3	9	9	
60	60	20	10						
1	2	4	7						
1	2	4	–						
1	2	–	–						
7	2	–	–						
10	2	–	–						

Min cost =

Transportation Problems

13 Sep, 2023

unbalanced TP

⇒ If in a Trans Prob ~~to~~ sum of all available quantities is not equal to the sum of requirements

ie $\sum_{i=1}^{n} a_i \neq \sum_{j=1}^{n} b_j$

then such problem is called unbalanced Transportation problem.

Working Rule

whenever $\sum a_i \leq \sum b_j$, Introduce a dummy source in the Transportable table. The cost of transportation from this dummy source to any destination are all set = 0. The availability at this dummy source is assumed to be equal to the difference $\sum b_j - \sum a_i$, $\sum b_j \leq \sum a_i$

Ques

	D1	D2	D3	ai
O1	4	3	2	10
O2	2	5	0	13
O3	3	8	6	12
bj	8	5	4	

$\because \sum b_j - \sum a_i$
$= 35 - 17 = 18$

Soln $\sum a_i = 35 \neq 17 = \sum b_j$

So, it is unbalanced TP.

	D1	D2	D3	Dummy	
O1	4	3	2	0	10
O2	2	5	0	0	13
O3	3	8	6	0	12
	8	5	4	18	35

Now it is balanced

4 ⑧	3 ②	2	0	10	2	2	2	3
2	5 ③	0 ④	⑥ 0	13	0	0	0	5
3	8	6	⑫ 0	12	3	–	–	–
8	5	4	18					
1	2	2	0					
2	2	2	0					
–	2	2	–					
–	2	–	0					

no. of allocation
= 6 = 3+4−1

non deg soln

min cost = 4×8 + 3×2 + 5×3 + 0×4 + 0×6 + 12
= 32 + 6 + 15 = 53

ui & vj

					ui
	4	3			−2
		5	0	0	0
				0	0
vj	6	5	0	0	

dij

		4	2
−4			
−3	3	6	

not optimal

4 ⑧ −	3 + ②	2	0
2 +θ	5 − ③	0 ④	0 ⑥
3	8	6	0 ⑫

min {8−θ, 3−θ}
θ = 3

4 ⑤	3 ⑤	2	0
2 ③	5	0 ④	0 ⑥
3	8	6	0 ⑫

⇒

20 + 15 + 6 = 41

u_i & v_j

				u_i
4	3			2
2		0	0	0
			0	0
v_j 2	1	0	0	0

d_{ij}

		0	-2
	4		
1	7	6	

Not Optimal

4 ⑤ $-\theta$	3 ⑤	2	0 $+\theta$
2 ③ $+\theta$	5	0 ④	0 ⑥ $-\theta$
3	8	6	0 ⑫

Min $\{5-\theta, 6-\theta\}$

$\theta = 5$

4	3 ⑤	2	0 ⑤
2 ⑧	5	0 ④	0 ①
3	8	6	0 ⑫

$15 + 16 = \boxed{31}$ Ans $\boxed{\text{Opt Ans}}$

u_i & v_j

				u_i
	3		0	0
2		0	0	0
			0	0
v_j 2	3	0	0	

d_{ij}

2		2	
	2		
1	5	6	

all $d_{ij} > 0$

Min opt sol^n = 31

Maximisation Transportation Problem

Step 1 convert the given problem into minimization by replacing each element of the transportation table its difference from the maximum element of the table.

Q

				a_i
20	21	16	18	10
17	28	14	16	9
29	23	19	20	7
b_j 6	10	4	5	

$\Sigma a_i = 26$, $\Sigma b_j = 25$

unbalanced.

20	21 ⑩	16	18	0	10	16 2 2 –
17 ⑥	28	14 ③	16	0	9	14 2 2 2
29	23	19 ①	20 ⑤	0 ①	7	19 1 1 1
6	10	4	5	1	26	
3	2	2	2	0		
3	2	2	2	–		
–	2	2	2	–		
–	–	5	4	–		

no. of all. = 6 < m+n-1 = 3+5-1 = 7.

degenerate.

Sol^n $21 \times 10 + 17 \times 6 + 14 \times 3 + 9 \times 1 + 20 \times 5$

= ~~210+~~ 463

Test for optimality

Dooay

27/Sept./23.

UNIT-3.

GAME THEORY

Def^n: ① Game is defined as an activity b/w two or more persons involving activities by each person according to a set of rules at the end of which each person recieves some benefit or satisfaction or negative benefit.

② The set of rules defines the game. Going through the set of rules once by participants defines a play.

Characteristics of Game Theory

① Chance of Strategy : If in a game activities are determined by skill. It is said to be a game of strategy. If they are determined by chance, then it is the game of chance. In general, a game may involve game of strategy as well as a game of chance.

② No. of Persons : A game is called an 'n' person game if the no. of persons playing is 'n'. The person means an individual or a group aiming at a particular objective.

③ No. of Activities : These may be finite or infinite.

④ No. of alternatives (choices) available to each person: In a particular activity, no. of choices may be finite or infinite. A finite game has a finite no. of activities each involving a finite no. of choices, otherwise a game is said to be infinite.

⑤ Information to the players about the past activities of other players is completely available, partially available or not available at all.

⑥ Pay-off: A quantity measure of satisfaction, a person gets at the end of each play is called a payoff. It is a real valued function of the variable in a game.

Let v_i be the payoff to the player P_i, $1 \le i \le n$ in an 'n' person game.

If $\sum_{i=1}^{n} v_i = 0$, then the game is said to be a zero sum game.

Some - Basic Definitions

1. Competitive Game: Competitive games are those in which players play against one another and where one player winning means another player loses. The games like chess, poker etc. have the characteristics of competition and are played according to some definite rules. Games theory provides optimal solutions to such games, assuming that each of the players wants to maximize his profit or minimize his loss.

OR

A competitive situation is called a competitive game if it has the following four properties:

(i) There are finite no. (n) of competitors (called players) s.t. $n \ge 2$. In case, $n=2$, it is called a two-person game and in case $n>2$, it is referred to as a n-person game.

(ii) Each player has a list of finite no. of possible activities.

(iii) A play is said to occur when each player choose one of his activities. The choices are assumed to be made simultaneously i.e., no player knows the choice of the other until he has decided on his own.

(iv) Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. -ve gain implies the loss of same amount.

Zero sum and non-zero sum Game :

2. Zero Sum and non-zero sum Game: If the players make payments only to each other i.e. the loss of one is the gain of others, and nothing comes from outside, the competitive game is said to be zero sum game.

A game which is not zero-sum is called a non-zero sum game.

3. Strategy : A strategy of a player has been loosely defined as a rule for decision-making in advance of all the plays by which he decides the activities he should adopt.
In other words, a strategy for a given player is the set of rules which specifies which ~~course~~ of the available course of action he should make at each play.
The strategy may be of two kinds :

(i) Pure strategy :- The pure strategy is a decision rule always to select a particular course of action.

(ii) Mixed - strategy :- The mixed strategy is a selection among pure strategies with fixed probabilities.

4. Two - person zero sum (or Rectangular) Games : A game with only two players is called a two-person zero sum game. If the losses of one player are equivalent of the other, so that the sum of their net gains is zero.

5. Pay - off Matrix : Suppose the player A has 'm' activities and player B has 'n' activities, then a pay-off matrix can be formed by adopting the following rules :

(i) Row designations for each matrix are activities available to player A.

(ii) Column designations for each matrix are activities available to player B.

(iii) Cell entry Vij is the payment to the player A in A's payoff matrix when A select the activity i & B choose the activity B j.

(iv) With a zero-sum two person game, the cell entry in the player B playoff matrix will be negative of the corresponding cell entry Vij in the player A playoff matrix. So that sum of the payoff matrices for player A & player B is zero.

lar space, completely
Ur lemma, Statement of

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{Player B}} \\ & 1 & 2 & \cdots & j & \cdots & n \\ \hline 1 & V_{11} & V_{12} & \cdots & V_{1j} & \cdots & V_{1n} \\ 2 & V_{21} & V_{22} & \cdots & V_{2j} & \cdots & V_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ i & V_{i1} & V_{i2} & \cdots & V_{ij} & \cdots & V_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ m & V_{m1} & V_{m2} & \cdots & V_{mj} & \cdots & V_{mn} \end{array}$$

player A

A's payoff matrix

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{Player B}} \\ & 1 & 2 & \cdots & j & \cdots & n \\ \hline 1 & -V_{11} & -V_{12} & \cdots & -V_{1j} & \cdots & -V_{1n} \\ 2 & -V_{21} & -V_{22} & \cdots & -V_{2j} & \cdots & -V_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ i & -V_{i1} & -V_{i2} & \cdots & -V_{ij} & \cdots & -V_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ m & -V_{m1} & -V_{m2} & \cdots & -V_{mj} & \cdots & -V_{mn} \end{array}$$

player A

B's payoff matrix

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Maximin Principle :

For a player A, the minimum value in each row represents the least gain (pay off) to him if he chooses his particular strategy. These are written in the matrix by row minima. He will then select the strategy that give the largest gain among the row minima value. This choice of player A is called ~~maxima~~ maximin principle and the corresponding gain is called the maximin value of the game.

Minimax Principle :

For player B who is assumed to be the looser, the maximum value in each column the maximum loss to him, if he chooses this particular strategy. These are written in the payoff matrix by column maxima. He will select the strategy that give min. loss among the column max. value. The choice of player B is called minimax principle and the corresponding value (loss) is called the minimax value of the game.

Optimal Strategy :

If the maximin value is equal to minimax value, then game is said to have a saddle (equillibrium) point and the corresponding strategy is called optimal strategy.

Value of the game :

This is the expected pay off at the end of the game when each player uses his optimal strategy i.e. the amount of payoff matrix at the equillibrium point. A game may have more than one saddle point. A game with no saddle point are solved by choosing strategies with fixed probabilities.

Remark : ① The value of the game satisfy the equation Maximin value $\leq V \leq$ minimax value.

② A game is said to be a fair game if the lower (maximin) and upper (minimax) value of the games are equal and both equals to zero.

③ A game is said to be strictly determinable if the lower (maximin) and upper (minimax) value of the game are equal & both equal to the value of the game.

Rules to determine saddle point

① Select the lowest element in each row of the payoff and write them under row minima heading, then select the largest element among these elements and enclose it in a rectangle.

② Select the largest element in each column of the payoff matrix and write them under column maxima heading. Then select the lowest element among these element and enclose it in a circle.

③ Find out the element i.e, same in the circle as well as rectangle and mark the position of such element in the matrix. This element represent the value of the game and it is called the saddle point or equilibrium point.

Ques

	Player B			
Player A	B_1	B_2	B_3	Row Minimum
A1	-1	2	$\boxed{-2}$	$\boxed{-2}$
A2	6	4	-6	-6
Column Maximum	6	4	$\bigcirc\!\!\!\!-2$	

saddle point.

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Minimax Criteria of Optimility

Imp. Defin

It states that, if a player list the worst possible outcomes of all his potential strategies, he will choose that strategy to be the most suitable for him which corresponds to the best of these worst outcomes. Such a strategy is called an optimal strategy.

Ques Consider two-person zero sum game matrix, which represents payoff to the player A. Find the optimal strategy (if any).

$$\begin{array}{cc|ccc} & & & B & \\ & & I & II & III \\ & I & -3 & -2 & 6 \\ A & II & 2 & 0 & 2 \\ & III & 5 & -2 & 4 \end{array}$$

Soln: Find saddle point :

		B			
		I	II	III	Row Min.
	I	-3	-2	6	-3
A	II	2	0	2	$\boxed{0}$ maximum.
	III	5	-2	4	-2
Col Max.		5	$\bigcirc\!\!\!\!0$ minimum	6	

Max Mini = 0 = MiniMax.

Saddle point exist here.

Optimal strategy for Player A = II
optimal strategy for Player B = II

Ques Consider the following game

B

		1	2	3	Row Min.
	1	3	-4	8	-4
A	2	-8	5	-6	-8
	3	6	-7	6	-7
Col Max.		6	5	8	

} -4 Maximum

5 minimum

Max mini = ~~8~~ -4   mini Pe max
Mini max = ~~eq~~ 5   max Pe mini

which in diff.

Ques Find the range of values of p & q which will declare the entry (2,2) a saddle point for the following table.

Player B

		1	2	3	
Player A	1	2	4	5	2
	2	10	7	q	7
	3	4	p	6	4
		10	7	~~8~~q	

p=5
q=8

$q \geq 7$ & $p \leq 7$

Ques. The payoff matrix of a game is given. Find the solution of the game to the player A and player B.

B

		I	II	III	IV	V	Row min
	I	-2	0	0	5	3	-2
	II	3	2	1	2	2	①
A	III	-4	-3	0	-2	6	-4
	IV	5	3	-4	2	-6	-6
	Col max.	5	3	$\boxed{1}$	5	6	

} 1

1

The best strategy for player A is II
& The best strategy of the player B is III.

The value of the game is 2 to Player A
& -1 to player. B.

Practice Work

Ques Find the saddle point and hence solve the following games:

Ques1

B

		B₁	B₂	B₃	Row Min
A	A_1	15	2	3	2
	A_2	6	5	7	5 ← max.
	A_3	-7	4	0	-7
	Col Max.	15	5	7	

↑min

Max mini = 5 = Minimax. ⇒ Saddle point exist.
optimal strategy for A & B = (A_2, B_2)

gular space,
Urysohn's lemma, Statem

Qus 2

$$\begin{array}{c|cccc|c} & & & B & & \\ & B_1 & B_2 & B_3 & B_4 & \text{Row min} \\ \hline A_1 & 1 & 7 & 3 & 4 & 1 \\ A \quad A_2 & 5 & 6 & 4 & 5 & 4 \leftarrow \\ A_3 & 7 & 2 & 0 & 3 & 0 \\ \hline \text{Col Max} & 7 & 7 & 4 & 5 & \\ & & & \uparrow & & \end{array}$$

Minimax = 4 = Maxmini
⇒ saddle point exist.

Optimal strategy for player A = A_2
Optimal strategy for player B = B_3.

Qus 3

$$\begin{array}{c|cccc|c} & & B & & & \\ & I & II & III & IV & \text{Row min} \\ \hline I & -5 & 2 & 1 & 20 & -5 \\ A \quad II & 5 & 5 & 4 & 6 & 4 \leftarrow \\ III & 4 & -2 & 0 & -5 & -5 \\ \hline \text{Col max} & 5 & 5 & 4 & 20 & \\ & & & \uparrow & & \end{array}$$

Minimax = 4 = maximini
⇒ saddle point exist.

Optimal strategy for player A = II
Optimal strategy for player B = III

Qus 4

$$\begin{array}{c|ccccc|c} & & & B & & & \\ & I & II & III & IV & V & \text{Row min} \\ \hline I & 9 & 3 & 1 & 8 & 0 & 0 \\ A \quad II & 6 & 5 & 4 & 6 & 7 & 4 \leftarrow \\ III & 2 & 4 & 4 & 3 & 8 & 2 \\ IV & 5 & 6 & 2 & 2 & 1 & 1 \\ \hline \text{Col max} & 9 & 6 & \rightarrow 4 & 8 & 8 & \end{array}$$

Maxmini = 4 = minimax.
⇒ saddle point exist

Optimal strategy for player A = II
Optimal strategy for player B = III

Qus 5

$$\begin{array}{c|ccc|c} & & B & & \\ & I & II & III & \text{Row min} \\ \hline A \quad I & 6 & 8 & 6 & 6 \leftarrow \\ II & 4 & 12 & 2 & 2. \\ \hline \text{Col max.} & 6 & 12 & 6 & \\ & \uparrow & & \uparrow & \end{array}$$

Maxmini = 6 = Minimax
There exist two saddle points.

Optimal strategies are (I, I) & (I, III).

Qus 6

$$\begin{array}{c|ccc|c} & & B & & \\ & C_1 & C_2 & C_3 & \text{Row min} \\ \hline R_1 & 3 & 0 & -3 & -3 \\ A \quad R_2 & 2 & 3 & 1 & 1 \leftarrow \\ R_3 & -4 & 2 & -1 & -4 \\ \hline \text{Col Max.} & 3 & 3 & 1 & \\ & & & \uparrow & \end{array}$$

Maxmini = 1 = Minimax.
⇒ saddle point exist.

Optimal strategy for player A = R_2
Optimal strategy for player B = C_3

Ques 7 Solve the games whose payoff matrices are given by.

(a)

		Player B		Row min
	1	2	1	1 ←
Player A	0	-4	-1	-4
	1	3	-2	-2
Col Max.	1 ↑	3	1 ↑	

optimal strategies are (1, 1) or (1, III).

~~maximin~~ V = 1 for A
V = -1 for B.

(b)

		B					Row min
		I	II	III	IV	V	
A	I	7	8	4	6	8	4 ←
	II	-8	6	1	9	6	-8
	Col Max	7	8	4 ↑	9	8	

~~optimal strategies are (1, II) or (9,91)~~

Minimax = 4 = Maximin.

Optimal strategy for A = I
" " " B = III

value of game for A = 4
value of game for B = -4.

Ques 8. Solve the games whose payoff matrices are given below :

(a)

			B		Row min
		B_1	B_2	B_3	
A	A_1	-3	-1	6	-3
	A_2	2	0	2	0 ←
	A_3	5	-2	-4	-4
	Col max.	5	0 ↑	6	

Max min = 0 = minimax.
optimal strategy = (A_2, B_2)
V = 0

(b)

			B		Row min
		B_1	B_2	B_3	
A	A_1	15	2	3	2.
	A_2	6	5	7	5 ←
	A_3	-7	4	0	-7
	Col max.	15	5 ↑	7	

maxmin = 5 = minimax
optimal strategy = (A_2, B_2)
V = 5.

Ques 9 For what value of 'a' the game with the following pay-off matrix is strictly determinable?

			B		Row Min
		B_1	B_2	B_3	
A	A_1	a	6	2	a
	A_2	-1	a	-7	-7
	A_3	-2	4	a	-2
	Col Max.	a	6	2	

$\left.\begin{matrix} a \\ -7 \\ -2 \end{matrix}\right\} a$

$\underbrace{a \quad 6 \quad 2}_{a}$

$-1 \leq a \leq 2$.

2x2 Games without Saddle Points

Consider a payoff matrix of player A.

$$\begin{array}{cc|cc|} & & \multicolumn{2}{c}{B} \\ & & y_1 & y_2 \\ \hline A & x_1 & V_{11} & V_{12} \\ & x_2 & V_{21} & V_{22} \\ \hline \end{array}$$

value of the Game to the player A is given by,

$$V = \frac{V_{11} \cdot V_{22} - V_{12} \cdot V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})}$$

& the formula for the optimal strategies can be found by using the results

$$\frac{x_1}{x_2} = \frac{V_{22} - V_{21}}{V_{11} - V_{12}}$$

$$\frac{y_1}{y_2} = \frac{V_{22} - V_{12}}{V_{11} - V_{21}}$$

$$x_1 = \frac{V_{22} - V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})}$$

$$x_2 = \frac{V_{11} - V_{12}}{V_{11} + V_{22} - (V_{12} + V_{21})}.$$

$$y_1 = \frac{V_{22} - V_{12}}{V_{11} + V_{22} - (V_{12} + V_{21})}$$

$$y_2 = \frac{V_{11} - V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})}$$

Ques. Find out the value of the game and optimal strategy:

$$\begin{array}{cc|cc|cl} & & \multicolumn{2}{c}{B} & & \\ & & y_1 & y_2 & \text{Row Min} & \\ \hline A & x_1 & 8 & -3 & -3 & \multirow{2}{*}{$\left.\right\}-3$} \\ & x_2 & -3 & 1 & -3 & \\ \hline & \text{Col Max} & 8 & 1 & & \\ & & \multicolumn{2}{c}{\underbrace{\qquad\qquad}_{1}} & & \end{array}$$

∴ There does not exist saddle point.

$V_{11} = 8$, $V_{12} = -3$, $V_{21} = -3$, $V_{22} = 1$.

$$V = \frac{V_{11} \cdot V_{22} - V_{12} V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{(8 \times 1) - (-3 \times -3)}{(8+1) + (3+3)}$$

$$V = \frac{8 - 9}{9 + 6} = \frac{-1}{15}$$

$$x_1 = \frac{V_{22} - V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{1+3}{15} = \frac{4}{15}$$

$$x_2 = \frac{V_{11} - V_{12}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{8+3}{15} = \frac{11}{15}.$$

$$y_1 = \frac{V_{22} - V_{12}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{1+3}{15} = \frac{4}{15}.$$

$$y_2 = \frac{V_{11} - V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{8+3}{15} = \frac{11}{15}.$$

Arithmetic method for 2×2 Games

Step1: Find the difference of two numbers in column 1 and put it under the column 2, neglecting the negative sign, if occurs.

Step2: Find the difference of two numbers in column 2 and put it under column 1, neglecting the negative sign, if occurs.

Step3: Repeat the above two steps for the two rows also.

The value thus obtained are called the ~~odd ments~~ oddments. These are the frequencies with which the players must use their courses of action in their optimum strategies. The optimum strategy can be found by dividing sum of the differences.

Ques.

Player B

$$\text{Player A} \quad \begin{array}{cc} 8 & -3 \\ -3 & 1 \end{array}$$

Find out the value of game and optimal strategy.

Solⁿ:

$$\begin{array}{c|cc|c} & \text{Play B} & & \text{Row min} \\ \text{Player A} & 8 & -3 & -3 \\ & -3 & 1 & -3 \\ \hline \text{Col max} & 8 & 1 & \end{array} \quad \Big\} -3$$

$$\underbrace{8 \quad 1}_{1}$$

$$\begin{array}{c|cc|cl} & \text{Player B} & & & \\ \text{Player A} & 8 & -3 & 4 & \frac{4}{4+11} = \frac{4}{15} \\ & -3 & 1 & 11 & \\ \hline & 4 & 11 & & \frac{14}{4+11} = \frac{11}{15}. \end{array}$$

Optimum strategy for player A is $\left(\frac{4}{15}, \frac{11}{15}\right)$

and for player B is $\left(\frac{4}{15}, \frac{11}{15}\right)$

value of the game using player B oddments

$$V = \frac{(4 \times 8) + (11 \times -3)}{11+4} = \frac{32-33}{15} = \frac{-1}{15}$$

Ques. Find out the value of game and optimal strategy.

$$\begin{array}{cc|cc|c} & & \text{Player B} & & \\ & & \text{I} & \text{II} & \text{Row min} \\ \text{Player A} & \text{I} & -4 & 6 & -4 \\ & \text{II} & 2 & -3 & -3 \\ \hline & \text{Col max} & 2 & 6 & \end{array} \quad \Big\} -3$$

$$\underbrace{2 \quad 6}_{2}$$

Minimax ≠ Maxmini

∴ Saddle point does not exist.

$$\begin{array}{cc|cc|cc} & & \text{B} & & & \\ & & \text{I} & \text{II} & & \\ \text{A} & \text{I} & -4 & 6 & 5 & \frac{5}{15} \\ & \text{II} & 2 & -3 & 10 & \frac{10}{15} \\ \hline & & 9 & 6 & & \\ & & \frac{9}{15} & \frac{6}{15}. & & \end{array}$$

Optimal strategy for player A is $\left(\frac{5}{15}, \frac{10}{15}\right)$

& ~~player~~ B is $\left(\frac{9}{15}, \frac{6}{15}\right)$.

Value of the game ~~is~~ is given by.

$$V = \frac{(5\times -4) + (10\times 2)}{15} = \frac{-20+20}{15+15} =$$

$$V = 0$$

It means the game is fair.

Note: If the sum of row = sum of column, then value of game is same $\forall$.

Dominance Rules

Rule 1: If each element in one row, say r^{th} of the payoff matrix $[v_{ij}]$, is less than or equal to the corresponding element in the other row, say s^{th}, then the player A will never choose the r^{th} strategy. ~~In~~ In other ~~row~~ words, if $\forall$ $j = 1, 2, ---- n$ and $v_{rj} \le v_{sj}$, then the probability x_r of choosing r^{th} strategy will be zero. The value of the game and the non-zero choice of probabilities remain unaltered even if r^{th} row is deleted from the payoff matrix. Such r^{th} row is said to be dominated by the s^{th} row.

Rule 2: Following the similar arguments, if each element in one column, say C_r, is greater than or equal to the corresponding element in the other column, say C_s, then the player B will never use the strategy corresponding to column C_r. In this case, the column C_s dominates the column C_r.

Rule 3: Dominance need not be based on the superiority of pure strategies only. A given strategy can be dominated if it is inferior to an average of two or more other pure strategies. In general, if some convex linear combination of some rows dominates the i^{th} row, then the i^{th} row will be deleted. If the i^{th} row dominates the convex linear combination of some other rows, then one of the rows involving in the combination may be deleted. Similar arguments follow for columns also.

Rule 4: If (x_1, x_2) be the optimal strategy for the player A for the reduced game and (w_1, x_2) be the optimal strategy for the original game, then w_1 is the i^{th} place extension of x_1.

Rule 5: If (y_1, y_2) be the optimal strategy for the player B for the reduced game and (y_1, w_2) be the optimal strategy for the original game, then w_2 is the j^{th} place extension of y_2.

Rule 6: If the dominance holds strictly, then values of optimal strategies do coincide, and when the dominance does not hold strictly, then optimal strategies may not coincide.

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Ques

Player B

		I	II	III	Row min	
Player A	I	-4	6	3	-4	Max
	II	-3	-3	4	-3	-3
	III	2	-3	4	-3	
Col max		2	6	4		

Min 2

Soln

Minimax ≠ Maxmini

∴ No saddle point

Player B

		I	II
Player A	I	-4	6
	II	-3	-3
	III	2	-3

On comparing column I & III, elements of III are higher. So, neglect column III.

Player B

		I	II
Player A	I	-4	6
	III	2	-3

On comparing row II and III, elements of II are smaller. So, neglect II row.

Player B

		I	II	
Player A	I	-4	6	5/15
	III	2	-3	10/15
		9/15	6/15	

Row → kum wala delete

optimal strategy of Player A $\left(\frac{1}{3}, 0, \frac{2}{3}\right)$

optimal strategy of Player B $\left(\frac{3}{5}, \frac{2}{5}, 0\right)$

$$\text{value of game} = \frac{V_{11} V_{22} - V_{12} V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{12-12}{-15} = 0.$$

Ques

Player B

		I	II	III	IV	V	VI	Row min	
Player A	I	0	0	0	0	0	0	0	
	II	4	2	0	2	1	1	0	
	III	4	3	1	3	2	2	1	1
	IV	4	3	7	-5	1	2	-5	
	V	4	3	4	-1	2	2	-1	
	VI	4	3	3	-2	2	2	-2	
Col max.		4	3	7	3	2	2		

2

Maxmini ≠ Minimax

∴ No saddle point.

Row I & II both will be dominated by Row III.

Player B

		I	II	III	IV	V	VI
Player A	III	4	3	1	3	2	2
	IV	4	3	7	-5	1	2
	V	4	3	4	-1	2	2
	VI	4	3	3	-2	2	2

Col^{n} $\overline{V}$ dominated by Columns I, II & $\overline{VI}$

Player B

Player A

$$\begin{array}{c|ccc} & \overline{III} & \overline{IV} & \overline{V} \\ \hline \overline{III} & 1 & 3 & 2 \\ \overline{IV} & 7 & -5 & 1 \\ \overline{V} & 4 & -1 & 2 \\ \overline{VI} & 3 & -2 & 2 \end{array}$$

Row $\overline{VI}$ is dominated by $\overline{V}$.

Player B

Player A

$$\begin{array}{c|ccc} & \overline{III} & \overline{IV} & \overline{V} \\ \hline \overline{III} & 1 & 3 & 2 \\ \overline{IV} & 7 & -5 & 1 \\ \overline{V} & 4 & -1 & 2 \end{array}$$

Average of Player B strategies $\overline{III}$ & $\overline{IV}$ gives $\left\{2, 1, \frac{3}{2}\right\}$ which is superior to the player B's $\overline{V}^{th}$ pure strategy.

∴ delete the $\overline{V}^{th}$ strategy from the matrix.

Player B

Player A

$$\begin{array}{c|cc} & \overline{III} & \overline{IV} \\ \hline \overline{III} & 1 & 3 \\ \overline{IV} & 7 & -5 \\ \overline{V} & 4 & -1 \end{array}$$

Average of Player A strategies $\overline{III}$ & $\overline{IV}$ gives. $\{4, -1\}$ which is same as $\overline{V}^{th}$ strategy.

∴ delete Row $\overline{V}$ from the matrix.

Player B

Player A

$$\begin{array}{c|cc|l} & \overline{III} & \overline{IV} & \\ \hline \overline{III} & 1 & 3 & 12/14 = 6/7 \\ \overline{IV} & 7 & -5 & 2/14 = 1/7 \\ \hline & \frac{8}{14} & \frac{6}{14} & \\ & \frac{4}{7} & \frac{3}{7} & \end{array}$$

optimal strategy for player B is $\left(0, 0, \frac{4}{7}, \frac{3}{7}, 0, 0\right)$

and for player A is $\left(0, 0, \frac{6}{7}, \frac{1}{7}, 0, 0\right)$.

$$\text{value of game} = \frac{V_{11}V_{22} - V_{12}V_{21}}{V_{11}+V_{22}-(V_{12}+V_{21})}$$

$$= \frac{-5-21}{1-5-10} = \frac{-26}{-14} = \frac{13}{7}$$

11/10/23.

Ques

Player B

Player A

$$\begin{array}{c|ccccc|c} & B_1 & B_2 & B_3 & B_4 & B_5 & \text{Row}_{\text{min}} \\ \hline A_1 & 4 & 4 & 2 & -4 & -6 & -6 \\ A_2 & 8 & 6 & 8 & -4 & 0 & -4 \\ A_3 & 10 & 2 & 4 & 10 & 12 & 2 \\ \hline \text{Col max.} & 10 & 6 & 8 & 10 & 12 & \end{array} \quad 2 \text{ max}$$

$\underbrace{10 \quad 6 \quad 8 \quad 10 \quad 12}_{6 \rightarrow \text{Min}}$

Minimax ≠ Maxmini

∴ No Saddle point.

Row ~~B~~ I is dominated by Row III

$$\begin{array}{cc|ccccc} & & & & \text{Player B} & & \\ & & B_1 & B_2 & B_3 & B_4 & B_5 \\ & A_2 & 8 & 6 & 8 & -4 & 0 \\ \text{Player A} & A_3 & 10 & 2 & 4 & 10 & 12 \end{array}$$

B_1, ~~B_2~~ & B_3 ~~are~~ dominantes B_2.

$$\begin{array}{cc|ccc} & & & \text{Player B} & \\ & & B_2 & B_4 & B_5 \\ \text{Player A} & A_2 & 6 & -4 & 0 \\ & A_3 & 2 & 10 & 12. \end{array}$$

B_5 dominates B_4

$$\begin{array}{cc|cc|l} & & \text{Player B} & & \\ & & B_2 & B_4 & \\ \text{Player A} & A_2 & 6 & -4 & 8/18 = 4/9 \\ & A_3 & 2 & 10 & 10/18 = 5/9 \\ & & \frac{14}{18} & \frac{4}{18} & \\ & & = \frac{7}{9} & = \frac{2}{9} & \end{array}$$

optimal strategy for Player A is $\left(0, \frac{4}{9}, \frac{5}{9}, 0, 0\right)$

& Player B = $\left(0, \frac{7}{9}, 0, \frac{2}{9}, 0\right)$

16
+2

-4+2

Value of game = $\frac{V_{11} V_{22} - V_{12} V_{21}}{V_{11} + V_{22} - (V_{12} + V_{21})} = \frac{60 + 8}{18} = \frac{\cancel{68}^{34}}{\cancel{18}_{9}}$

$= \frac{34}{9}$.

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18 | October | 2023.

Graphical Method [for (2xn) and (mx2) games]

Consider the (2xn) plan assuming that the game doesn't have a saddle point

$$\begin{array}{ccc|cccc|} & & & & \text{Player B.} & & \\ & & & B_1 & B_2 & - - - - - - & B_n \\ & & & y_1 & y_2 & - - - - - - & y_n \\ \hline & A_1 & x_1 & V_{11} & V_{12} & - - - - - & V_{1n} \\ \text{Player A} & A_2 & 1-x_1 & V_{21} & V_{22} & - - - - - & V_{2n} \\ \hline \end{array}$$

Since the player A has two strategies, it follows that $x_2 = 1 - x_1$, where $x_1 \geq 0$, $x_2 \geq 0$.

∴ This shows that, player A's expected payoff varies linearly with x_1

B's Pure Strategies	A's expected Payoff.
B_1	$V_{11} x_1 + V_{21} (1-x_1)$
B_2	$V_{12} x_1 + V_{22} (1-x_1)$
⋮	⋮
B_n	$V_{1n} x_1 + V_{2n} (1-x_1)$

According to the maxmini criteria for mixed strategy games. The player A should select, the value of x_1 so as to maximize his minimum expected payoff. This may be done by plotting the straight lines

$E(x_1) = (V_{11} - V_{21})\, x_1 + V_{21}$

$E(x_2) = (V_{12} - V_{22})\, x_1 + V_{22}$

$\vdots$

$E(x_n) = (V_{1n} - V_{2n})\, x_1 + V_{2n}.$

The lowest boundaries of these lines will give the minimum expected payoff as function of x_1.

The Highest point on this lowest boundary would then give the maxmini expected payoff and the optimum value of x_1 is x_1^*.

Ques Solve the 2x3 games graphically,

Player B

$$\text{Player A} \quad \begin{array}{c|ccc|} & I & II & III \\ \hline I & 1 & 3 & 11 \\ II. & 8 & 5 & 2 \\ \hline \end{array}$$

Sol^n:

$E(x_1) = -7x_1 + 8$

$E(x_2) = -2x_1 + 5$

$E(x_3) = 9x_1 + 2$

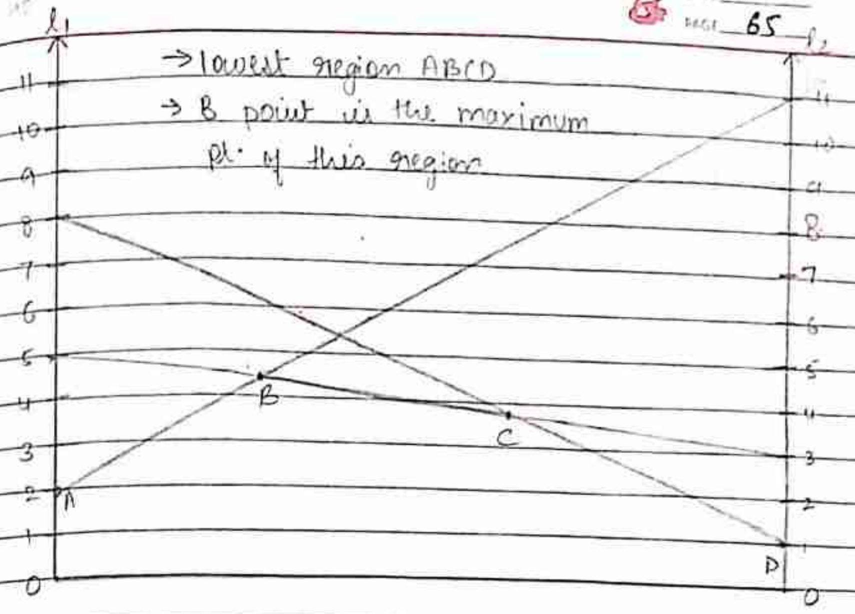
B

$$A \quad \begin{array}{c|c|c|c} & II & III & \\ \hline I & 3 & 11 & 3/11 \\ II & 5 & 2 & 8/11 \\ \hline & 9/11 & 2/11 & \end{array}$$

$A \rightarrow \left(\frac{3}{11}, \frac{8}{11}\right)$

$B \rightarrow \left(0, \frac{9}{11}, \frac{2}{11}\right)$

Value of game $\Rightarrow \dfrac{6-55}{5-16} = \dfrac{49}{11}$.



→ lowest region ABCD

→ B point is the maximum pt. of this region

Ques Solve the game graphically.

$$\text{Player A} \quad \begin{array}{c|cc|} & \overset{y}{I} & \overset{1-y}{II} \\ \hline I & 2 & 4 \\ II & 2 & 3 \\ III & 3 & 2 \\ IV & -2 & 6 \\ \hline & \text{line 2} & \text{line 1} \end{array}$$

Player B

$E(y_1) = 2y_1 + (1-y_1)\,4 = 2y_1 + 4 - 4y_1$
$= 4 - 2y_1$

$E(y_2) = 2y_1 + (1-y_1)\,3 = 2y_1 + 3 - 3y_1$
$= 3 - y_1$

$E(y_3) = 3y_1 + (1-y_1)\,2 = 3y_1 + 2 - 2y_1$
$= 2 + y_1$

$E(y_4) = -2y_1 + (1-y_1)\,6 = -2y_1 + 6 - 6y_1$
$= 6 - 8y_1$

B

$$A \quad \begin{array}{c|cc|c} & I & II & \\ \hline I & 2 & 4 & 1/3 \\ III & 3 & 2 & 2/3 \\ \hline & 2/3 & 1/3 & \end{array}$$

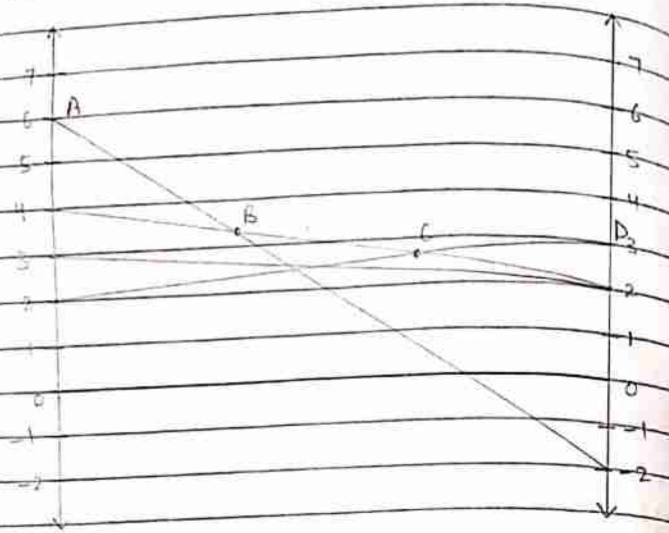
$A \rightarrow \left(\frac{1}{3}, 0, \frac{2}{3}, 0\right)$

$B \rightarrow \left(\frac{2}{3}, \frac{1}{3}\right)$

Value of game $= \dfrac{4-12}{4-7} = \dfrac{8}{3}$.

~~# Sequencing~~

~~Suppose there are~~



Sequencing

Suppose there are n jobs (1, 2, 3, ----, n) each of which has to be processed one at a time at each of m machines [A, B, C, -----]. The order of processing each job through machine is given [for eg :- Job 1 is processed through the machine A, C, --- in this order].

The time ~~the~~ each job must require on each machine is known. The problem is to find a sequence among $(n!)^m$ number of all possible sequences for processing the jobs so that total elapsed time ∀ the jobs will be minimum.

Mathematically,

let A_i = Time for Job i on machine A

B_i = Time for Job i on machine B

T = Time for start of 1st Job to the completion of last job.

Then the problem is to determine for each machine a sequence of jobs $(i_1, i_2, --- i_n)$ where $i_1, i_2, --- i_n$ is the permutation of integer which will minimize T.

Ideal time on a machine : This is the time for which a machine remain ideal during the total elapsed time. The notation $[X_{ij}]$ shall be used to denote the ideal time of machine j b/w the end of the $(i-1)^{th}$ job & the start of the i^{th} job.

Total elapsed time : This is the time b/w starting the 1st job & completing the last job. This include the Ideal time if exist. It is denoted by T.

No passing rule : This rule means that the passing is not allowed i.e., the same of order of Job is maintained over each machine.

If each of the n jobs to be processed through 2 machines A & B in this order AB then this rule mean that each job will go to the machine A first and then to B.

Processing 'n' jobs through 2 machines

(i) The problem can be described as only 2 machine A & B are involved.

(ii) Each job is processed in the order AB (say)

(iii) The exact or expected processing time $A, A_2, ---, A_n$ and $B_1, B_2, ---B_n$ are known.

Job(i)

		1	2	3	---	i	---	n
Machine	A	A_1	A_2	A_3	---	A_i	---	A_n
	B	B_1	B_2	B_3	---	B_i	---	B_n

Johnson's Algorithm :

Step1 : Examine the A_i's & B_i's for $i = 1, 2, ---, n$ & find out minimum of $[A_i, B_i]$ $\forall i$.

Step2 : (i) If this min be A_K for $i=K$, do the K^{th} job first of all.

(ii) If this min be B_r for $i=r$, do the r^{th} job last of all.

Step3 : (i) If there is a tie for minima i.e. $A_K = B_r$ process the K^{th} Job first & r^{th} job in the last.

(ii) If the tie for, the minimum occurs among A_i's, select the job corresponding to the min. B_i's & process it first of all. *[B max vala aage se jayega]

(iii) If the tie for minimum occurs among the B_i's 's, select the job corresponding to the min. A_i's. & process it in the last. [*A me jo max hoga vo last me]

Step4 : Cross out the jobs already assigned and repeat step1 to 3 arranging the jobs next to first or next to last untill all the jobs have been assigned.

Que. There are 5 jobs each of which must go through the 2 machines AB. Processing time are given in the table : (Processing time in hours).

Jobs.

		1	2	3	4	5
Machine	A	5	1	9	3	10
	B	2	6	7	8	4

Determine a sequence for 5 jobs that will minimize the elapsed time T. Also calculate the total ideal time for the machine in this period.

Solⁿ: It is seen that the smallest processing time is 1 for job 2 at machine A so,

2	4	3	5	1

← optimal sequence.

Further we will calculate the elapsed time corresponding to min. using individual processing time.

Optimal job sequence.	Machine A		Machine B	
	Time IN	Time OUT	Time IN	Time OUT
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	30

Ideal time for A = 30 - 28 = 2 hr
Ideal time for B = 1 ~~+~~ 1+(23-22) = 3 hr

Min. elapsed time = 30 hr.

Ques. In a factory there are 6 jobs to perform each of which should go through 2 machines in the order AB. The processing time (in hrs) for the jobs are given here, you have to find the sequence for performing the jobs that would minimize the total elapsed time 'T'. Also find the value of T.

Jobs		1	2	3	4	5	6
Machine	A	1	3	8	5	6	3
	B	5	6	3	2	2	10

Sol^n. Optimal sequence :-

1	6	2	3	4	5
J_1	J_6	J_2	J_3	J_4	J_5

Minimum elapsed time :-

Optimum Sequence	Machine A		Machine B	
	Time IN	Time OUT	Time IN	Time OUT
1	0	1	1	6
6	1	4	6	16
2	4	7	16	22
3	7	15	22	25
4	15	20	25	27
5	20	26	27	29

Min. Total Time elapsed = 29 hr.

Ideal time :-
for machine A = (29-26) = 3 hr
for machine B = 1 hr

Ques

Books	1	2	3	4	5	6
Machine A	30	120	50	20	90	100
Machine B	80	100	90	60	30	10

Sol^n: Optimal sequence :-

4	1	3	2	5	6
J_4	J_1	J_3	J_2	J_5	J_6

optimum Sequence	Machine A		Machine B	
	Time IN	Time OUT	Time IN	Time OUT
4	0	20	20	80
1	20	50	80	160
3	50	100	160	250
2	100	220	250	350
5	220	310	350	380
6	310	410	410	420

Total time elapsed (T) = 420 hr.

Ideal time :-

for machine A = (420 − 410) = 10 hr

for machine B = (30 + 20) = 50 hr.

Player : A competitor in the game is known as player.

Strategy : A set of alternative courses of action available to player based on advanced knowledge is known as strategy.

Pure Strategy : If the player selects same strategy each time.

Mixed strategy : If the player selects his course of action in accordance with fixed probability.

Optimum Strategy : A course of action which puts the player in the most preferred position.

Two-person zero-sum game : When only two players are involved in the game and if the gain made by one player is equal to loss of the other.

Fair Game : If the value of the game is zero.

Solution of the Game having Saddle point (Pure Strategy)

If the max-min value equals to Min-max value, then the game is said to have saddle point & corresponding strategy is called optimum strategy.

Ques Solve the following 2-person zero sum game with following 3x2 payoff matrix of player A.

$$\begin{array}{c|cc|c|l} \text{Player A} \backslash \text{Player B} & B_1 & B_2 & \text{Row min} & \text{maxmini} \\ \hline A_1 & 9 & 2 & 2 & \\ A_2 & 8 & \textcircled{6} & \textcircled{6} & 6 \text{ Max} \\ A_3 & 6 & 4 & 4 & \\ \hline \text{Col max} & 9 & \textcircled{6} & & \\ \text{minimax} & \multicolumn{2}{c|}{6 \to \text{Min}} & & \end{array}$$

Min max = 6 = Max min
∴ saddle point exist

optimal strategy for A is A_2
optimal strategy for B is B_2

value of game = 6.

Ques Solve the Game,

$$\begin{array}{c|cccc|c|l} \text{Player A} \backslash \text{Player B} & B_1 & B_2 & B_3 & B_4 & \text{Row min} & \text{Maxmini} \\ \hline A_1 & -5 & 2 & 0 & 7 & -5 & \\ A_2 & 5 & 6 & \textcircled{4} & 8 & \textcircled{4} & 4 \text{ Max} \\ A_3 & 4 & 0 & 2 & -3 & -3 & \\ \hline \text{Col max} & 5 & 6 & \textcircled{4} & 8 & & \\ \text{minimax} & \multicolumn{4}{c|}{4 \text{ Min}} & & \end{array}$$

Minimax = 4 = Maxmin
∴ saddle point exist.

optimum strategy for A = A_2 $\quad$ V= 4
for B = B_3

Ques Solve the Game.

$$\begin{array}{c|ccc|c|l} A \backslash B & I & II & III & \text{Row min} & \\ \hline I & 2 & \textcircled{-1} & 8 & \textcircled{-1} & \\ II & -4 & -3 & 4 & -4 & -1 \text{ Max} \\ III & -8 & -4 & 0 & -8 & \\ IV & 1 & -6 & -2 & -6 & \\ \hline \text{col max} & 2 & \textcircled{-1} & 8 & & \\ & \multicolumn{3}{c|}{\text{Min } -1} & & \end{array}$$

Maxmin = −1 = Min max

optimal strategy for A = ~~A~~ I
for B = II
V= −1

Ques Find range of p & q that will make the payoff matrix given below.

$$\begin{array}{c|ccc|c|l} A \backslash B & 1 & 11 & 111 & \text{R min} & \\ \hline 1 & 2 & 4 & 7 & 2 & \\ 11 & 10 & \textcircled{7} & q & \textcircled{7} & 7 \text{ Max} \\ 111 & 4 & p & 8 & 4 & \\ \hline \text{Cmax} & 10 & \textcircled{7} & 8 & & \\ & \multicolumn{3}{c|}{7 \text{ min.}} & & \end{array}$$

Maxmin = Minmax = 7
∴ Saddle pt. exist.

value of game = 7.

optimal solⁿ = (11, 11)

$7 \leq q \leq 8$
$4 \leq p \leq 7$

Ques Solve the following 2-person zero sum game with following 3x2 payoff matrix of player A.

$$\begin{array}{cc|cc|cl}
 & & \text{Player } B & & & \\
 & & B_1 & B_2 & \text{Row min} & \\
\hline
 & A_1 & 9 & 2 & 2 & \\
\text{Player } A & A_2 & 8 & \textcircled{6} & \textcircled{6} & 6 \text{ Max} \quad \text{maximin} \\
 & A_3 & 6 & 4 & 4 & \\
\hline
\text{Col max} & & 9 & \textcircled{6} & & \\
\text{minimax} & & \underbrace{\quad\quad}_{6 \to \text{Min}} & & &
\end{array}$$

Minmax = 6 = Maxmin
∴ saddle point exist

optimal strategy for A is A_2
optimal strategy for B is B_2

value of game = 6.

Ques Solve the Game,

$$\begin{array}{cc|cccc|cl}
 & & & \text{Player } B & & & & \\
 & & B_1 & B_2 & B_3 & B_4 & \text{Row min} & \\
\hline
 & A_1 & -5 & 2 & 0 & 7 & -5 & \\
\text{Player } A & A_2 & 5 & 6 & \textcircled{4} & 8 & \textcircled{4} & 4 \text{ Max} \quad \text{Maxmin} \\
 & A_3 & 4 & 0 & 2 & -3 & -3 & \\
\hline
\text{Col max} & & 5 & 6 & \textcircled{4} & 8 & & \\
\text{minimax} & & & & 4 \text{ Min} & & &
\end{array}$$

Minimax = 4 = Maxmin
∴ saddle point exist.

optimum strategy for A = A_2 V = 4
for B = B_3

Ques Solve the Game.

$$\begin{array}{cc|ccc|cl}
 & & & B & & & \\
 & & I & II & III & \text{Row min} & \\
\hline
 & I & 2 & \textcircled{-1} & 8 & \textcircled{-1} & \\
A & II & -4 & -3 & 4 & -4 & -1 \text{ Max} \\
 & III & -8 & -4 & 0 & -8 & \\
 & IV & 1 & -6 & -2 & -6 & \\
\hline
\text{col Max} & & 2 & \textcircled{-1} & 8 & & \\
 & & & \text{Min } -1 & & &
\end{array}$$

Maxmin = -1 = Min max

optimal strategy for A = ~~A~~ I
for B = II
V = -1

Ques Find range of p & q that will make the payoff matrix given below.

$$\begin{array}{cc|ccc|cl}
 & & & B & & & \\
 & & I & II & III & R\,\text{min} & \\
\hline
 & I & 2 & 4 & 7 & 2 & \\
A & II & 10 & \textcircled{7} & q & \textcircled{7} & 7 \text{ Max} \\
 & III & 4 & p & 8 & 4 & \\
\hline
\text{Cmax} & & 10 & \textcircled{7} & 8 & & \\
 & & & 7 \text{ min.} & & &
\end{array}$$

Maxmin = Minmax = 7
∴ Saddle pt. exist.

Value of game = 7.
optimal solⁿ = (II, II)

$7 \leq q \leq 8$
$4 \leq p \leq 7$

Game Without Saddle Point (Mixed strategy)

A mixed strategy game can be solved by following method:

① Algebraic method
② Graphical method.
③ Linear Programming Method.

Algebraic Method

Consider the 2-person zero-sum game with following payoff matrix.

$$\begin{array}{cc|cc|} & & \multicolumn{2}{c}{\text{Player B}} \\ & & \text{I} & \text{II} \\ \text{Player A} & \text{I} & a_{11} & a_{12} \\ & \text{II} & a_{21} & a_{22} \end{array}$$

let x_1, x_2 be prob. of using 1st & 2nd strategy by player A s.t $x_1 + x_2 = 1$

$\Rightarrow x_2 = 1 - x_1$

& let y_1, y_2 be prob. of using 1st & 2nd strategy by player B s.t $y_1 + y_2 = 1$

$\Rightarrow y_2 = 1 - y_1$

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})} \qquad y_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})}$$

$$v = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$$

Ques Solve the following game whose pay-off matrix is

$$\begin{array}{c|cc|c} & \multicolumn{2}{c}{\text{Player B}} & \\ \text{Player A} & 3 & -2 & 3/8 \\ & 2 & 5 & 5/8 \\ \hline & \frac{7}{8} & \frac{1}{8} & \end{array}$$

optimal strategy of A $= \left(\frac{3}{8}, \frac{5}{8}\right)$

" " of B $= \left(\frac{7}{8}, \frac{1}{8}\right)$

$$V = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

$$V = \frac{15+4}{8-0} = \frac{19}{8}$$

or

$$V = 3 \times \frac{3}{8} + 2 \times \frac{5}{8}$$

$$= \frac{9}{8} + \frac{10}{8} = \frac{19}{8}$$

Ques Find out the value of game

$$\begin{array}{cc|cc|l} & & \multicolumn{2}{c}{B} & \\ & & B_1 & B_2 & \\ A & A_1 & 4 & -4 & 8/16 = 1/2 \\ & A_2 & -4 & 4 & 8/16 = 1/2 \\ & & \frac{8}{16} & \frac{8}{16} & \\ & & =\frac{1}{2} & =\frac{1}{2} & \end{array}$$

optimal strategy of A $= \left(\frac{1}{2}, \frac{1}{2}\right) =$ O.S. of B.

$$v = 4 \times \frac{1}{2} - 4 \times \frac{1}{2} = 0$$

$\therefore$ Fair Game

Ques In game of matching coins, player A wins Rs 8/- if both coins show heads & Rs 1/- if both coin show. Tails, Player B wins Rs 3/- when coins do not match. Given the choice of being Player A or B, which would you choose and what would be your strategy.

Player B

		H	T	
Player A	H	8	-3	4/15
	T	-3	1	11/15
		$\frac{4}{15}$	$\frac{11}{15}$	

optimal strategy for A = $\left(\frac{4}{15}, \frac{11}{15}\right)$

for B = $\left(\frac{4}{15}, \frac{11}{15}\right)$

$$V = 8\times\frac{4}{15} - \frac{3\times 11}{15} = \frac{32}{15} - \frac{33}{15}$$

$$V = \frac{-1}{15}$$

Dominance Rule :

R-min
C-max

① Row Dominance

min row delete

B

		C_1	C_2	C_3
A	R_1	4	6	-5
	R_2	5	6	-3

Delete R_1

Ⓘ Column Dominance.

max col Delete.

	C_1	C_2
R_1	4	2
R_2	0	-1
R_3	-3	-3

Delete C_1

When comparison becomes impossible, we take average of 2 rows & compare with remaining row → same for column.

Modified Row Dominance

B

		B_1	B_2
A	A_1	2	4
	A_2	5	0
	A_3	0	8
		2.5	4

Delete A_1

Modified Column Dominance

B

		B_1	B_2	B_3	
A	A_1	8	15	1	8
	A_2	1	-1	4	1.5

Delete B_1

Ques Solve the following game using the Dominance concept.

Player B

		B_1	B_2	B_3
	A_1	4	5	8
Player A	A_2	6	4	6
	A_3	4	2	4

saddle pt doesn't exist

Step1 Delete A_3 b'coz each element of A_2 is greater than or equal to each element of A_3

Player B

		B_1	B_2	B_3
Player A	A_1	4	5	8
	A_2	6	4	6

Step2 Delete B_3 b'coz each element of B_3 is greater than or equal to each element of B_2

Player B

$$\text{Player A} \begin{array}{c|cc|c} & B_1 & B_2 & \\ \hline A_1 & 4 & 5 & 2/3 \\ A_2 & 6 & 4 & 1/3 \\ \hline & \frac{1}{3} & \frac{2}{3} & \end{array}$$

optimal strategy for A = $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$

for B = $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$

$$V = 2\times\frac{4}{3} + 6\times\frac{1}{3} = \frac{8}{3}+\frac{6}{3} = \frac{14}{3}$$

Que Solve the following 2-person zero sum game based on the concept of Dominance.

$$\begin{array}{cc|ccc|cl} & & & B & & & \\ & & I & II & III & \text{Row min} & \\ \hline & I & -4 & 6 & 3 & -4 & \\ A & II & -3 & -3 & 4 & -3 & -3 \text{ max} \\ & III & 2 & -3 & 4 & -3 & \\ \hline \text{Col max} & & 2 & 6 & 4 & & \\ & & & \underbrace{\hphantom{2 \quad 6 \quad 4}}_{2 \text{ min}} & & & \end{array}$$

Minmax ≠ Maxmin

∴ No saddle point.

Step 1: Delete row II b'coz each element of row III is greater or equal to row II

$$\begin{array}{cc|ccc|} & & & B & \\ & & I & II & III \\ \hline & I & -4 & 6 & 3 \\ A & III & 2 & -3 & 4 \end{array}$$

Step 2: Delete column III b'coz each element of col III is greater or equal to col I.

$$\begin{array}{cc|cc|l} & & I & II & \\ \hline A & I & -4 & 6 & 5/15 \to 1/3 \\ & III & 2 & -3 & 10/15 \to 2/3 \\ \hline & & \frac{9}{15} & \frac{6}{15} & \\ & \to & \frac{3}{5} & \frac{2}{3} & \end{array}$$

Optimum strategy for A = $\left(\frac{1}{3}, 0, \frac{2}{3}\right)$

for B = $\left(\frac{3}{5}, \frac{2}{3}, 0\right)$

$$V = \frac{1}{3}\times -4 + \frac{4}{3} = \frac{-4}{3}+\frac{4}{3} = 0$$

∴ Fair Game

Que Using the Dominance concept, obtain the optimal strategy.

$$\begin{array}{cc|ccccc|cl} & & & & B & & & & \\ & & I & II & III & IV & V & \text{Row min} & \\ \hline & I & 2 & 4 & 3 & 8 & 4 & 2 & \\ A & II & 5 & 6 & 3 & 7 & 8 & 3 & 6 \text{ Max} \\ & III & 6 & 7 & 9 & 8 & 7 & 6 & \\ & IV & 4 & 2 & 8 & 4 & 3 & 2 & \\ \hline \text{Col max} & & 6 & 7 & 9 & 8 & 8 & & \\ & & & & 6 \text{ Min} & & & & \end{array}$$

Maxmin = Minmax

∴ Saddle point exist

Step 1: Delete row I b'coz each element of row III is greater or equal to row I

Step 2: Delete row IV b'coz each element of row III is greater than or equal to row IV.

$$\begin{array}{cc|ccccc|} & & & & B & & \\ & & I & II & III & IV & V \\ A & II & 5 & 6 & 3 & 7 & 8 \\ & III & 6 & 7 & 9 & 8 & 7 \end{array}$$

Step 3: Delete Colm II & V as each element of col II & V is greater or equal to col I

$$\begin{array}{cc|ccc|c|} & & & B & & \\ & & I & III & IV & \cancel{V} \\ A & II & 5 & 3 & 7 & \cancel{8} \\ & III & 6 & 9 & 8 & \cancel{7} \end{array}$$

Step 4: Delete Col IV b'coz each element of col IV is greater or equal to col I.

$$\begin{array}{cc|cc|c|} & & & B & \\ & & I & III & \cancel{IV} \\ A & II & 5 & 3 & \cancel{8} \\ & III & 6 & 9 & \cancel{7} \end{array}$$

Since saddle point exist

∴ ~~Value~~ V = 6

optimal strategy (III, I)

Prob. for Player A = (0,0,1,0)
Prob. for Player B = (1,0,0,0,0)

Ques Deduce the following game using the concept of Dominance & solve it.

$$\begin{array}{cc|cccc|c} & & & B & & & \text{Row min} \\ & & I & II & III & IV & \\ & I & 3 & 2 & 4 & 0 & 0 \\ A & II & 3 & 4 & 2 & 4 & 2 \\ & III & 4 & 2 & 4 & 0 & 0 \\ & IV & 0 & 4 & 0 & 8 & 0 \\ \hline & \text{Col Max} & 4 & 4 & 4 & 8 & \end{array}$$

2 Max

4 Min

Max min ≠ Minimax
∴ no saddle point.

Step 1: Delete row I b'coz each element of row III is greater than or equal to row I.

$$\begin{array}{cc|cccc|} & & & B & & \\ & & I & II & III & IV \\ & II & 3 & 2 & 4 & 0 \\ A & III & 4 & 2 & 4 & 0 \\ & IV & 0 & 4 & 0 & 8 \end{array}$$

Step 2: Delete row II b'coz each element of row III is greater or equal to each element of row II.

$$\begin{array}{cc|cccc|} & & & B & & \\ & & I & II & III & IV \\ A & III & 4 & 2 & 4 & 0 \\ & IV & 0 & 4 & 0 & 8 \end{array}$$

Step 3: Delete Col I

$$\begin{array}{cc|ccc|} & & & B & \\ & & II & III & IV \\ A & III & 2 & 4 & 0 \\ & IV & 4 & 0 & 8 \end{array}$$

Step 4: Delete Col II

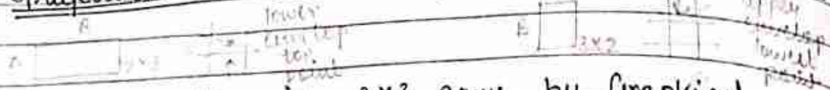
$$\begin{array}{cc|cc|ccc} & & B & & & & \\ & & III & IV & & & \\ A & III & 4 & 0 & 8/12 & \rightarrow & 2/3 \\ & IV & 0 & 8 & 4/12 & \rightarrow & 1/3 \end{array}$$

$$\text{value of game} = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$$

$$= \frac{32-0}{12-0} = \frac{+68}{63} = \boxed{\frac{8}{3} = V}$$

optimum strategy for A = $\left(0, 0, \frac{2}{3}, \frac{1}{3}\right)$ = 0.1 for B

Graphical Method : Useful for 2×M & N×2 order



Que Solve the following 2×3 game by Graphical method.

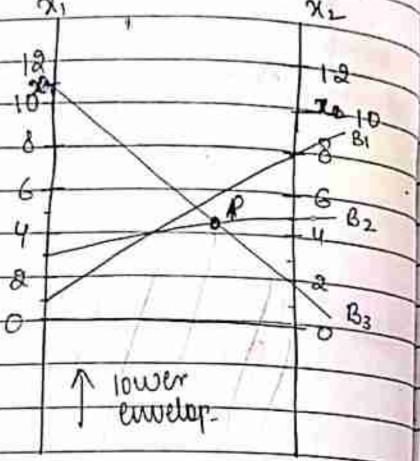
$$\begin{array}{c|ccc} & & B & \\ & I & II & III \\ \hline A\ x_1\ I & 1 & 3 & 11 \\ x_2\ II & 8 & 5 & 2 \end{array}$$

$x_2 = 1-x_1$

$E(x_1) = -7x_1 + 8$

$(x_2) = -2x_1 \overset{+5}{+9}$ P is the point of

$(x_3) = 9x_1 + 2$ maximum in the lower envelop.



$$\begin{array}{c|cc|c} & B & & \\ & II & III & \\ \hline A\ I & 3 & 11 & 3/11 \\ II & 5 & 2 & 8/11 \\ \hline & \frac{9}{11} & \frac{2}{11} & \end{array}$$

$A \rightarrow \left(\frac{3}{11}, \frac{8}{11}\right)$

$B \rightarrow \left(0, \frac{9}{11}, \frac{2}{11}\right)$

$$V = 3 \times \frac{3}{11} + 5 \times \frac{8}{11} = \frac{9}{11} + \frac{40}{11} = \frac{49}{11}.$$

Que Reduce the following 2×n game to 2×2 by Graphical method & solve it.

$$\begin{array}{c|ccccc} & & & B & & \\ & I & II & III & IV & V \\ \hline A\ x_1\ I & 2 & -1 & 5 & -2 & 6 \\ x_2\ II & -2 & 4 & -3 & 1 & 0 \end{array}$$

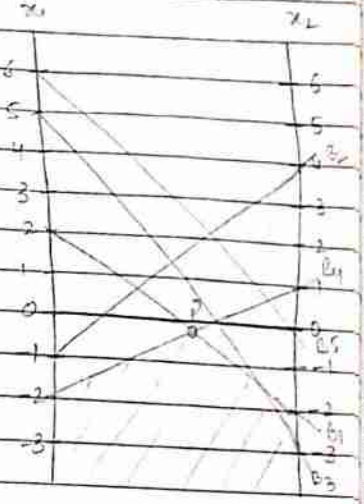
$E(x_1) = 4x_1 - 2$

$E(x_2) = -5x_1 + 4$

$E(x_3) = 8x_1 - 3$

$E(x_4) = -3x_1 + 1$

$E(x_5) = 6x_1 + 0.$



P is the point of maximum in the lower envelop.

$$\begin{array}{c|cc|c} & B & & \\ & I & IV & \\ \hline A\ I & 2 & -2 & 3/7 \\ II & -2 & 1 & 4/7 \\ \hline & \frac{3}{7} & \frac{4}{7} & \end{array}$$

$A \rightarrow \left(\frac{3}{7}, \frac{4}{7}\right)$ $B \rightarrow \left(\frac{3}{7}, 0, 0, \frac{4}{7}, 0\right)$

$$V = \frac{6}{7} - \frac{8}{7} = \frac{-2}{7}$$

Que Solve the following game using Graphical Method.

$$\begin{array}{c|cc} & B_1\ y_1 & B_2\ y_2 \\ \hline A_1 & -6 & 7 \\ A_2 & 4 & -5 \\ A_3 & -1 & -2 \\ A_4 & -2 & 5 \\ A_5 & 7 & -6 \end{array}$$

$E(y_1) = -13y_1 + 7$

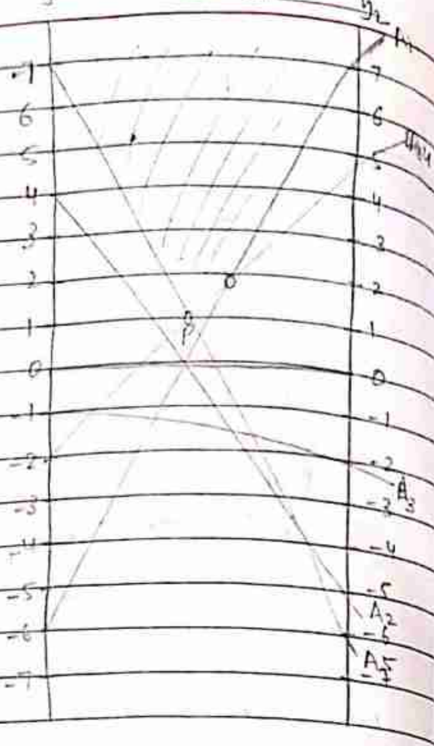
$E(y_2) = 9y_1 - 5$

$E(y_3) = y_1 - 2$

$E(y_4) = -7y_1 + 5$

$E(y_5) = 13y_1 - 6$

P is the lowest point in the upper envelop.



	B_1	B_2	
A_4	-2	5	13/20
A_5	7	-6	7/20
	$\frac{11}{20}$	$\frac{9}{20}$	

$A \rightarrow \left(0, 0, 0, \frac{13}{20}, \frac{7}{20}\right)$

$B \rightarrow \left(\frac{11}{20}, \frac{9}{20}\right)$

$V = -\frac{26}{20} + \frac{49}{20} = \frac{23}{20}$

Que: Use the dominance rule to reduce the following game in either 2XM or NX2 game & then solve by graphical method.

		B			
	I	II	III	IV	Row min
A I	19	6	7	5	5
II	7	3	14	6	3
III	12	8	18	4	4
IV	8	7	13	-1	-1
Col max	19	8	18	6	

5 Max

6 min

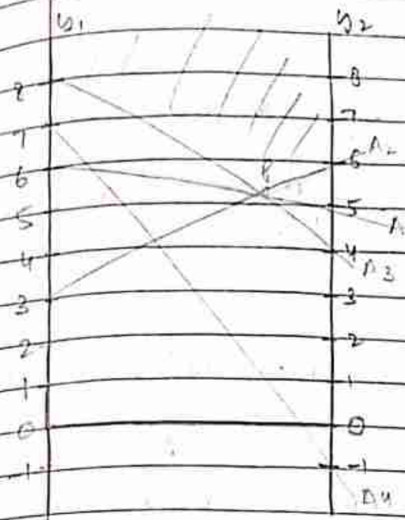
Maxmin ≠ Minmax

∴ no saddle point

~~Delete row IV~~

		B		
	I	II	III	IV
A I	19	6	7	5
II	7	3	14	6
III	12	8	18	4

Delete Col I & III & row IV



	B	
	II	IV
A I	6	5
II	3	6
III	8	4

P is the lowest point in upper envelop and is the pt. of intersection of A_1 & A_2.

∴

	B		
	II	IV	
A I	6	5	3/4
II	3	6	1/4
	$\frac{1}{4}$	$\frac{3}{4}$	

$A \rightarrow \left(\frac{3}{4}, \frac{1}{4}, 0, 0\right)$

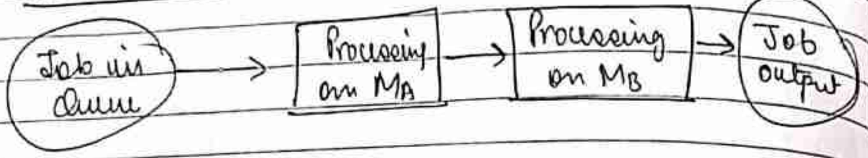
$B \rightarrow \left(0, \frac{1}{4}, 0, \frac{3}{4}\right)$

$V = \cancel{\frac{18}{4}}\ 6 \times \frac{3}{4} + \frac{3}{4} = \frac{21}{4}$

Sequencing :

Types of problems :
- ① n Jobs on a single machine
- ⑪ n Jobs on two machines
- (iii) n Jobs on three machines
- (iv) n Jobs on m machines
- (v) two jobs on m machine.
- (vi)

Johnson's Rule (n Jobs on two machine)



Ques Consider the processing time (in min) of 5 Jobs each of which must go through the two machines M_1 & M_2 in order $M_1 M_2$. Find the sequence for job that minimize total elapsed time. Also find Ideal time for each Machine. ~~Show the sequence on~~ Show the sequence on Gantt chart.

Job	J_1	J_2	J_3	J_4	J_5
M_1	5	1	9	3	10
M_2	2	6	7	8	4

optimum Sequence :→

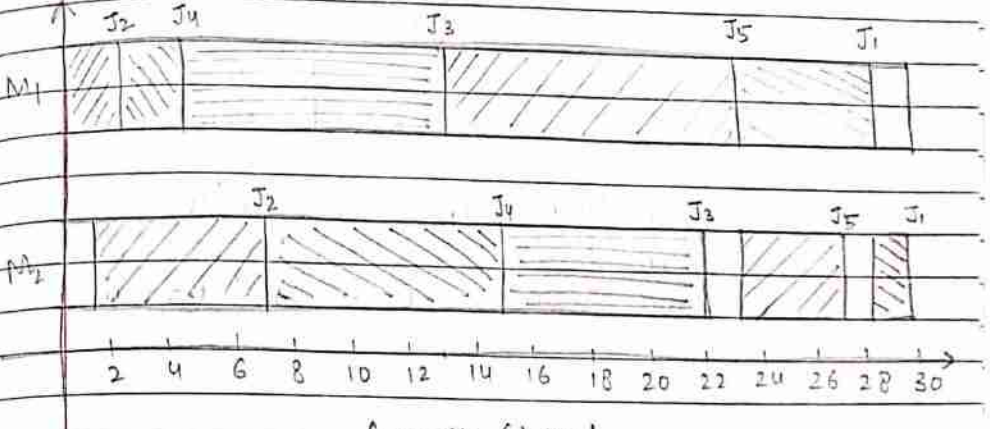
2	4	3	5	1
J_2	J_4	J_3	J_5	J_1

Minimum elapsed time :

Optimum Sequence	Machine 1		Machine 2	
	Time IN	Time OUT	Time IN	Time OUT
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	30

Total time elapsed = 30 hr

Ideal time for M_1 = (30 − 28) = 2 hr
Ideal time for M_2 = (1+1+1) = 3 hr



Gantt Chart

Ques In a factory there are 6 jobs to be perform. each of which should go through two machine A & B in order A B. The processing time (in min) for jobs are given below. Find the sequence for job that minimize total elapsed time. Also find ideal time for each machine.

→ choose where difference is max.

Job	1	2	3	4	5	6	
A	1	3	8	5	6	3	26
B	5	6	3	2	2	10	28

optimum sequence : | 1 | 6 | 2 | 3 | 4 | 5 |

Min. Time elapsed :

Optimum sequence	Machine A		Machine B	
	Time IN	Time OUT	Time IN	Time OUT
1	0	1	1	6
6	1	4	6	16
2	4	7	16	22
3	7	15	22	25
4	15	20	25	27
5	20	26	27	29

Total time elapsed = 29 min
Ideal time for A = (29-26) = 3 min
Ideal time for B = 1 min